# A mass formula for baryon resonances

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Light-baryon resonances with u,d, and s quarks only can be classified using the non-relativistic quark model. When we assign to baryon resonances with total angular momenta J intrinsic orbital angular momenta L and spin S we make the following observations: plotting the squared masses of the light-baryon resonances against these intrinsic orbital angular momenta L, $\Delta^*$'s with even and odd parity can be described by the same Regge trajectory. For a given L, nucleon resonances with spin S=3/2 are approximately degenerate in mass with $\Delta$ resonances of same total orbital momentum L. To which total angular momentum L and S couple has no significant impact on the baryon mass. Nucleons with spin 1/2 are shifted in mass; the shift is - in units of squared masses - proportional to the component in the wave function which is antisymmetric in spin and flavor. Sequential resonances in the same partial wave are separated in mass square by the same spacing as observed in orbital angular momentum excitations. Based on these observations, a new baryon mass formula is proposed which reproduces nearly all known baryon masses.

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Phenomenological analyses of transition energies between energy levels of bound systems can provide deep insight into the underlying dynamics. The Balmer formula demonstrated that the interpretation of the hydrogen atom must be simple; the formula was given long before Bohr derived the famous model which bares his name. Our understanding of nucleon-nucleon interactions was boosted by the discovery that the magic numbers in nuclear physics can be understood in terms of a nuclear shell structure in the presence of strong spin-orbit forces. And the analogy of the charmonium states with those of positronium atoms provided not only evidence for the existence of a new flavor but was also the final proof for the reality of quarks. In this letter we propose a new mass formula for light baryon resonances which reproduces 81 of the 82 masses of baryons with known spin and parity given by the Particle Data Group [1]. We assume that the baryon mass spectrum is due to the dynamics of three constituent quarks and that a confinement interaction gives rise to linear Regge trajectories [2]. The study aims to identify the dominant residual interactions between the constituent quarks. The mass formula reads [3]:

$$\mathrm{M}^2 = \mathrm{M}_\Delta^2 + \frac{\mathrm{n_s}}{3}\cdot \mathrm{M}_\mathrm{s}^2 + \mathrm{a}\,(\mathrm{L}+\mathrm{N}) - \mathrm{s_i}\cdot \mathrm{I_{sym}} \qquad (1)$$

where
$$\mathrm{M}_\mathrm{s}^2 = \left(\mathrm{M}_\Omega^2 - \mathrm{M}_\Delta^2\right)$$
$$\mathrm{s_i} = \left(\mathrm{M}_\Delta^2 - \mathrm{M}_\mathrm{N}^2\right).$$

$n_s$ is number of strange quarks in the baryon. Mostly, baryon masses are assumed to increase linearly with the number of strange quarks. We use in (1) a quadratic dependence for sake of simplicity. The model has no parameter to account for the $\Lambda-\Sigma$ mass difference [4]. L is the total intrinsic orbital angular momentum, which we have to assign to each baryon resonance. N is the radial excitation quantum number; L+2N gives the harmonic-oscillator band. $\mathrm{M_N}, \mathrm{M_\Delta}, \mathrm{M_\Omega}$ are input parameters taken from PDG. $a = 1.142/\mathrm{GeV}^2$ is the Regge slope determined from the series of light (isoscalar and isovector) mesons with quantum numbers $\mathrm{J^{PC}} = 1^{--}, 2^{++}, 3^{--}, 4^{++}, 5^{--}, 6^{++}$. $\mathrm{I_{sym}}$ is the fraction of the wave function (normalized to the nucleon wave function) antisymmetric in spin and flavor. It depends on the SU(6) flavor wave function

$\mathrm{I_{sym}} = 1.0$ for S=1/2 and for octet baryons in 56-plets;
$\mathrm{I_{sym}} = 0.5$ for S=1/2 and for octet baryons in 70-plets;
$\mathrm{I_{sym}} = 1.5$ for S=1/2 and for singlet baryons;
$\mathrm{I_{sym}} = 0$ otherwise.

For a quantitative comparison of our mass formula (1) with the experimental masses of the light-baryon resonances, central values and their uncertainties need to be defined. As mass value of a resonance we take - when given - the central value of the interval suggested by the Particle Data Group. We do not take experimental uncertainties of the mass determination into account, since they are only given for well established resonances. Instead, we use a simple estimate based on contributions from the hadronic width and a model error. It is well known that hadronic effects like opening thresholds, virtual decays and mixing with other states may result in mass shifts. To account for these effects we allow for an error of one quarter of the hadronic width of a resonance. A constant model error of 30 MeV is added quadratically to give the total error $\sigma_\mathrm{M}$. Since the measured widths show a wide spread and are often rather inaccurate, we use $\Gamma = Q/4$ as width estimate where $Q$ is the largest kinetic energy accessible in hadronic decays of the resonance. Our estimated uncertainties vary (for N and $\Delta$ resonances) from 40 MeV at 1500 MeV to 120 MeV at 3 GeV. Note that experimental uncertainties in the mass determination are often in the same range.

According to eq. (1), the squared baryon masses depend linearly on the intrinsic orbital angular momentum

L. Measured is of course only the total angular momentum J. We identify multiplets with intrinsic spin 3/2 using the following criteria: first, we identify 'stretched' states with J=L+S; L=0,1,..,6 and S=3/2, i.e. resonances with quantum numbers $J^P = 3/2^+, 5/2^-, 7/2^+, 9/2^-, 11/2^+, 13/2^-, 15/2^+$. These are shown in Table I in the last column. Omitted are the decuplet ground states (L=0) which also fall into this category.

In our eq. (1) we do not account for spin-orbit forces, assuming they are small or vanishing. Therefore, we collect all resonances of a spin 3/2 multiplet from a mass window (here we chose $\pm\sqrt{2}\sigma_M$) around the same stretched state requiring the same parity. In the non-relativistic quark model we expect single resonances for L=0 (the ground states), triplets for L=1 and quartets for higher L. The multiplet structure is clearly visible in Table I, even though the multiplets are not complete, supporting our assumption in eq. (1) of small or vanishing spin-orbit forces.

TABLE I: Baryon resonances assigned to S=3/2 multiplets. Baryon masses depend only weakly on the orientation of the spin relative to the orbital angular momentum: spin-orbit forces are small ($\vec{L}\cdot\vec{S} \sim 0$). Missing states are marked by a - sign.

| L | J=L-3/2 | J=L-1/2 | J=L+1/2 | J=L+3/2 |
|---|---|---|---|---|
| 1 | | $N_{1/2^-}(1650)$ | $N_{3/2^-}(1700)$ | $N_{5/2^-}(1675)$ |
| 1 | | $\Delta_{1/2^-}(1900)$ | $\Delta_{3/2^-}(1940)$ | $\Delta_{5/2^-}(1930)$ |
| 1 | | $\Lambda_{1/2^-}(1800)$ | - | $\Lambda_{5/2^-}(1830)$ |
| 1 | | $\Sigma_{1/2^-}(1750)$ | - | $\Sigma_{5/2^-}(1775)$ |
| 2 | - | $N_{3/2^+}(1900)$ | $N_{5/2^+}(2000)$ | $N_{7/2^+}(1990)$ |
| 2 | $\Delta_{1/2^+}(1910)$ | $\Delta_{3/2^+}(1920)$ | $\Delta_{5/2^+}(1905)$ | $\Delta_{7/2^+}(1950)$ |
| 2 | - | - | $\Lambda_{5/2^+}(2110)$ | $\Lambda_{7/2^+}(2020)$ |
| 2 | - | $\Sigma_{3/2^+}(2080)$ | $\Sigma_{5/2^+}(2070)$ | $\Sigma_{7/2^+}(2030)$ |
| 3 | - | $N_{5/2^-}(2200)$ | $N_{7/2^-}(2190)$ | $N_{9/2^-}(2250)$ |
| 3 | - | $\Delta_{5/2^-}(2350)$ | - | $\Delta_{9/2^-}(2400)$ |
| 4 | - | $\Delta_{7/2^+}(2390)$ | $\Delta_{9/2^+}(2300)$ | $\Delta_{11/2^+}(2420)$ |
| 5 | - | - | - | $\Delta_{13/2^-}(2750)$ |
| 6 | - | - | - | $\Delta_{15/2^+}(2950)$ |

Also quantitatively the comparison of our mass formula (1) with the light-baryon resonances with spin assignment S=3/2 (see Table I) is doing well: We get a $\chi^2 = 23.6$ for 31 data points.

We now turn to a discussion of spin 1/2 resonances. The lowest-mass spin-1/2 states have intrinsic L=0, positive parity and belong to an octet in the 56-plet representation. We now search for doublets of nearly mass-degenerate states with J=L±1/2. Doublets are observed for L=1,2, and 3; for larger L only one state with L+1/2 is known. The spin 1/2 states are collected in Fig. 1, grouped according to their SU(6) classification. The positive parity octet states have a shift in squared mass relative to the Regge trajectory of $0.657 \pm 0.035$ GeV$^2$. This value is compatible with the $\Delta_{3/2^+}(1232)$-N mass square difference (0.636 GeV$^2$). The negative-parity octet resonances undergo a mass shift of $(0.311 \pm 0.023)$ GeV$^2$, consistent with 1/2 of the $\Delta_{3/2^+}(1232)$-N mass square difference. We have also included the $N_{5/2^-}(2200)$ and $N_{7/2^-}(2190)$ from Table I here, since their intrinsic spin assignment (S=3/2 or 1/2) is ambiguous.

The $\Lambda_{1/2^-}(1405)$ and $\Lambda_{3/2^-}(1520)$ with their low masses are assigned to the SU(6) singlet system; the two states $\Lambda_{1/2^-}(1670)$ and $\Lambda_{3/2^-}(1690)$ form then the spin doublet of the 70-plet octet, and the $\Lambda_{1/2^-}(1800)$ and $\Lambda_{5/2^-}(1830)$ an incomplete spin-triplet (also belonging to a 70-plet). The $\Lambda_{7/2^-}(2100)$ is the lowest $\Lambda$ resonance with L=3; we assign it to the SU(6) singlet system because of its mass. The assignment is thus *ad hoc* as long as its octet partner (predicted by (1) at a mass of 2318 MeV) has not been found. These three singlet resonances have a large mass shift down from the Regge trajectory of $(0.942 \pm 0.059)$ GeV$^2$ or 3/2 times the $\Delta_{3/2^+}(1232)$-N mass difference.

There is one doublet of negative-parity $\Delta$ states, the $\Delta_{1/2^-}(1620)$ and $\Delta_{7/2^-}(1700)$. In addition we assign the $\Delta_{7/2^-}(2200)$ to the lowest-mass state with L=3 and S=1/2. It could also form a spin-3/2 quartet with the two other resonances $\Delta_{5/2^-}(2350)$ and $\Delta_{9/2^-}(2400)$. However, the $\Delta_{7/2^-}(2200)$ does not fall into the $\pm\sqrt{2}\sigma_M$ corridor, hence we do not accept this as spin 3/2 state. The mean mass shift of the three remaining negative parity decuplet $\Delta$ states relative to the Regge trajectory is $(0.074 \pm 0.103)$ GeV$^2$ and compatible with zero.

Summarizing the S=1/2 states, we observe a reasonable agreement with the experimental masses with eq. (1) resulting in a $\chi^2$ contribution of 43.3 for 29 d.o.f. Especially the description of the deviation of (1) from the Regge trajectory by the additional symmetry term (last term in (1)) is nicely confirmed.

In Eq. (1), radial excitations are supposed to have the same mass spacing (per unit of excitation number) as orbital angular momentum excitations. In Table II we list resonances belonging to one partial wave, and their mass square differences. The differences are of the order of 1.1 GeV$^2$, not incompatible with the spacing per unit of L. The 14 new data points contribute $\delta\chi^2$=17.8. This observation is the basis of the L+N dependence in (1). Table II may contain some positive-parity resonances with L=2, S=3/2 with ambigous assignments.

So far, we have included all baryon resonances of known spin-parity except a few special cases. The $\Sigma_{3/2^-}(1580)$ has two stars in the PDG notation, but it is very low in mass and does possibly not exist [5]. We disregard this resonance. The $\Delta_{5/2^+}(2000)$ has two mass entries, at 1752 MeV and 2200 MeV, respectively. Using the higher mass value, it can be identified as radial

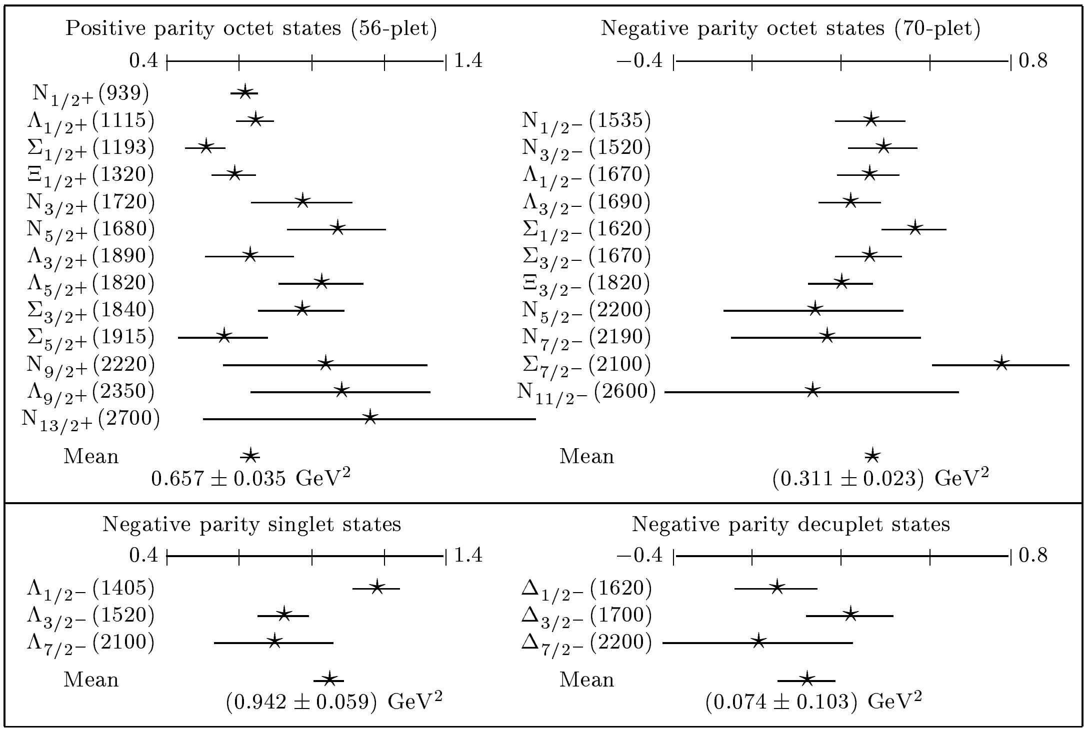


FIG. 1: Mass square shift (in $\mathrm{GeV}^2$) of spin-1/2 baryons w.r.t. the Regge trajectory $\mathrm{M}^2 = \mathrm{M}_\Delta^2 + \mathrm{n}_s/3 \cdot \mathrm{M}_s^2 + \mathrm{a}\,(\mathrm{L}+\mathrm{N})$ defined by baryons with S=3/2 (hyperfine splitting). The mass shifts scale as $1:1/2:3/2:0$ times $\mathrm{M}_\Delta^2 - \mathrm{M}_\mathrm{N}^2$ as we proposed in mass formula (1).

excitation of the $\Delta_{5/2^+}(1905)$ but this is clearly a speculation. There remain three states to be discussed, the $\Delta_{1/2^+}(1750)$ with one *, the $\Sigma_{1/2^+}(1660)$ (***) and the $\Sigma_{1/2^+}(1770)$ (*). Radial excitations of the $\Sigma_{3/2^+}(1385)$ are not necessarily in a 56-plet (then they have $3/2^+$); they can also fall into a 70-plet. In this case they have spin 1/2. The difference in squared mass between the $\Sigma_{1/2^+}(1770)$ and the $\Sigma_{3/2^+}(1385)$ is 1.21 $\mathrm{GeV}^2$, compatible with the other values in Table II. The $\Delta_{1/2^+}(1750)$ could be an analogous state; in this case the mass square difference is uncomfortably large, 1.54 $\mathrm{GeV}^2$; however the $\Delta_{1/2^+}(1750)$ is a one * resonance only. Likewise, the $\Sigma_{1/2^+}(1660)$ could be an octet radial excitation belonging to the SU(6) 70-plet. The mass difference to the first radial excitation in the 56-plet, possibly the $\Sigma(1560)$, is 0.322 $\mathrm{GeV}^2$, nearly identical to the other splittings between resonances belonging to the 56 or 70-plet. So, while the resonances discussed in this last paragraph cannot be used to validate the mass formula (1), they are nevertheless consistent with it when appropriate quantum numbers are assigned. These four states and the two remaining decuplet ground states (the $\Delta$ and $\Omega$ masses are used as input parameters) contribute $\delta\chi^2$=7.1.

In summary we compared 81 resonances to their masses according to the values summarized by the Particle Data Group and obtain a $\chi^2 = 91.7$ for 78 degrees of freedom.

We now discuss consequences for our understanding of the baryon mass spectrum. The mass formula (1) contains the orbital angular momentum as decisive quantity for baryon masses. The orbital angular momentum is the sum $\vec{\mathrm{L}} = \vec{l}_\rho + \vec{l}_\lambda$ of two orbital angular momenta associated with the two generalized coordinates of the three-particle system. All resonances are compatible with either $\mathrm{L} = l_\rho$ or $\mathrm{L} = l_\lambda$. A dynamical reason for this selection rule is not known; the question is related to the *missing resonance problem.*

Baryon resonances are classified according to the non-relativistic quark model. Doublets and quartets are clearly identified in the mass spectrum. The mass formula (1) does not include spin-orbit interactions. The proton spin puzzle underlines that our understanding of the dynamical role of the quark spin in baryons is not sufficient to exclude the possibility that spin-orbit interactions play no or little role in the baryon mass spectrum.

The second point resulting from this analysis is the energy gap of radial excitations. In the harmonic oscillator approximation, the first radial excitations are found in the second excitation band; the anharmonicity due to the confinement potential - supposed to be linear - shifts its mass down but not low enough to hit the mass of the Roper resonance at 1440 MeV or the $\Delta_{3/2^+}(1600)$. Table II shows a large number of recurrencies (17) which all give a small mass shift per increase in radial excita-

TABLE II: Excitations of baryon resonances having the same quantum numbers. The mean value per excitation is $(1.081 \pm 0.036)$ GeV$^2$, to be compared to the 1.142 GeV$^2$ from the fit to the meson Regge trajectory.

| Baryon | $\delta M^2$(GeV$^2$) | Baryon | $\delta M^2$(GeV$^2$) |
|---|---|---|---|
| $N_{1/2^+}(939)$ | | $\Delta_{3/2^+}(1232)$ | |
| $N_{1/2^+}(1440)$ | $1(1.18 \pm 0.11)$ | $\Delta_{3/2^+}(1600)$ | $1(1.04 \pm 0.15)$ |
| $N_{1/2^+}(1710)$ | $2(1.02 \pm 0.18)$ | $\Delta_{3/2^+}(1920)$ | $2(1.08 \pm 0.24)$ |
| $N_{1/2^+}(2100)$ | $3(1.18 \pm 0.29)$ | | |
| $\Lambda_{1/2^+}(1115)$ | | $\Sigma_{1/2^+}(1193)$ | |
| $\Lambda_{1/2^+}(1600)$ | $1(1.24 \pm 0.10)$ | $\Sigma_{1/2^{+?}}(1560)$ | $1(1.04 \pm 0.10)$ |
| $\Lambda_{1/2^+}(1810)$ | $2(0.98 \pm 0.15)$ | $\Sigma_{1/2^+}(1880)$ | $2(1.06 \pm 0.11)$ |
| $N_{1/2^-}(1535)$ | | $N_{3/2^-}(1520)$ | |
| $N_{1/2^-}(2090)$ | $2(1.01 \pm 0.31)$ | $N_{3/2^-}(2080)$ | $2(1.01 \pm 0.31)$ |
| $\Delta_{1/2^-}(1620)$ | | $\Delta_{3/2^-}(1700)$ | |
| $\Delta_{1/2^-}(1900)$ | $1(0.99 \pm 0.24)$ | $\Delta_{3/2^-}(1940)$ | $1(0.87 \pm 0.24)$ |
| $\Delta_{1/2^-}(2150)$ | $2(1.00 \pm 0.34)$ | | |
| | | $\Lambda_{3/2^-}(1670)$ | |
| | | $\Lambda_{3/2^-}(2325)$ | $2(1.31 \pm 0.27)$ |
| $\Sigma_{1/2^-}(1620)$ | | $\Sigma_{3/2^-}(1670)$ | |
| $\Sigma_{1/2^-}(2000)$ | $1(1.37 \pm 0.18)$ | $\Sigma_{3/2^-}(1940)$ | $1(0.97 \pm 0.17)$ |

tion number. Bijker *et al.* [6] have used an algebraic approach to describe baryon resonances. For them, the lowest recurrencies are one-phonon excitations and not two-phonon excitations as in the harmonic oscillator model.

Spin-spin interactions depend on the SU(6) symmetry of the baryon wave function. The symmetry term in (1) acts only for octet and singlet baryons (which have a component antisymmetric w.r.t. the exchange of two quarks) with spin 1/2 (which also has a component antisymmetric w.r.t. the exchange of two quarks). This latter component is reduced by a factor 2 in wave functions belonging to SU(3) octets within the SU(6) 70-plet. Of course, the overall wave functions in 56-plets and 70-plets have the same symmetry. Loosely speaking, in baryons with odd angular momentum, part of the antisymmetry is found in the spacial wave function. The $\Lambda$ resonances in the SU(6) singlet have negative parity, too. But now, all three quark pairs are antisymmetric in flavor w.r.t. exchange of two quarks. This gives the factor 3/2 enhancement of the symmetry contribution. Decuplet baryons or baryons with spin 3/2 do not have a wave function which is antisymmetric w.r.t. the exchange of two quarks both in spin and in flavor. They all fall onto the main Regge trajectory.

We thus need an interaction which gives rise to a mass shift proportional to the fraction of the wave function which is antisymmetric w.r.t. the exchange of two quarks both in spin and in flavor. This is a selection rule which holds for instanton-induced interactions [7]. The success of the eq. (1) provides therefore strong support that instanton-induced interactions play a decisive role for the spectrum of baryon resonances and are responsible for the hyperfine splitting. Interactions ascribed to one-gluon exchange can - at least to first order - be neglected.

The most model-discriminating masses are those of the negative-parity $\Delta$ resonances above 1.8 GeV. Capstick [8] finds them at about 2.1 GeV, Löring *et al.* [9] at 2.2 GeV. Bijker *et al.* [6] fit the data (with 11 parameters) and find 1.9 GeV, in agreement with data. In Glozman *et al.* [10] only the lower-mass states are calculated. The mass formula (1) yields 1.95 GeV. The least established $\Delta_{3/2^-}$ resonance is predicted to dominate the reaction $\gamma p \rightarrow \Delta_{3/2^-} \rightarrow \Delta_{3/2^+}(1232)\eta$ where the latter decay is in S-wave. Experiments along these lines are presently performed at ELSA [11].

We have shown that the spectrum of baryon resonances can be described successfully by a very simple mass formula. The squared masses increase linearly with the intrinsic orbital angular momentum between the constituent quarks, radial excitations have the same spacings as orbital excitations. Instanton-induced interactions reduce the masses whenever a component of the baryonic wave function is sensitive to their action. Gluon exchange leads to no significant contributions.

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[1] D. E. Groom et al. (Particle Data Group), Eur. Phys. J. **C15**, 1 (2000).
[2] A detailed analysis reveals slight non-linearities when hadronic mass shifts are neglected, see: A. Tang and J. W. Norbury, Phys. Rev. **D62**, 016006 (2000).
[3] E. Klempt, *Baryon resonances and strong QCD, in: BARYON2002, 9th Int. Conf. on the Structure of Baryon, Jefferson Laboratory, Newport News, VA, 2002.*
[4] The difference can be explained by assuming that the residual interactions between the constituent quarks are induced by instantons, and by introducing an isospin-dependence into these interactions, see: U. Loring, B. C. Metsch, and H. R. Petry, Eur. Phys. J. **A10**, 447 (2001).
[5] D. M. Manley et al., Phys. Rev. Lett. **88**, 012002 (2002).
[6] R. Bijker, F. Iachello, and A. Leviatan, Ann. Phys. **236**, 69 (1994).
[7] E. V. Shuryak and J. L. Rosner, Phys. Lett. **B218**, 72 (1989).
[8] S. Capstick and N. Isgur, Phys. Rev. **D34**, 2809 (1986).
[9] U. Löring, B. C. Metsch, and H. R. Petry, Eur. Phys. J. **A10**, 395 (2001).
[10] L. Y. Glozman, W. Plessas, K. Varga, and R. F. Wagenbrunn, Phys. Rev. **D58**, 094030 (1998).
[11] V. Crede, *Baryon spectroscopy, in: HADRON2001, 9th Int. Conf. on Hadon Spectroscopy, Protvino, Russia, 2001.*