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The physical meaning of scattering matrix singularities in coupled-channel formalisms*

BRAG 2007 Workshop summary

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Abstract. The physical meaning of bare and dressed scattering matrix singularities has been investigated. Special attention has been attributed to the role of the well-known invariance of the scattering matrix with respect to the field transformation of the effective Lagrangian. Examples of evaluating bare and dressed quantities in various models are given.

PACS. 11.80.Gw Multichannel scattering – 13.60.Le Meson production – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 14.20.Gk Baryon resonances with $S = 0$

This paper is a collection of different, sometimes conflicting standpoints presented at the BRAG 2007 premeeting of the NSTAR 2007 Workshop. It is organized in the following way: The introduction is written by A. Švarc, sect. 2.1 by S. Capstick, sect. 2.2 by C. Hanhart, sect. 3.1 by S. Scherer, sect. 3.2 by J. Gegelia, sect. 4.1 by M. Giannini and E. Santopinto, sect. 4.2 by T.-S.H. Lee, T. Sato and N. Suzuki and the conclusion by A. Švarc, S. Capstick and L. Tiator.

1 Introduction

Establishing a well-defined point of comparison between experimental results and theoretical predictions has for decades been one of the main issues in hadron spectroscopy, and the present status is still not satisfactory. Experiments, via partial wave (PWA) and amplitude analysis (AA), can give reliable information on scattering matrix singularities, while quark model calculations usually give information on resonant states spectrum in the first-order impulse approximation (bare/quenched mass spectrum). And these two quantities are by no means the same. Up to now, in the absence of a better recipe, these quantities have usually been directly compared, but the awareness has ripen that the clear distinction between the two has to be made. One either has to dress quark-model resonant states spectrum and compare the outcome to the experimental scattering matrix poles, or to try to take into account all self-energy contributions which are implicitly included in the measured scattering matrix pole parameters, make a model-independent undressing procedure and compare the outcome to the impulse approximation quark-model calculations. The first options seems to be feasible but complicated, but the latter one seems to be impossible due to very general field theory considerations.

We report on investigating both options.

An attempt how to unquench the constituent quark model of ref. [1], together with describing all accompanying complications, is presented. The procedure seems to be cumbersome, but straightforward.

The second option, undressing the experimentally obtained scattering matrix singularities, however, seems to be inherently model dependent due to very general

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arguments originating from the local field theory. A simple model, illustrating this claim, is presented.

In spite of looking entirely dissimilar, the problem of model-independent undressing of full scattering matrix singularities seems to be strongly correlated to the recent controversy whether the off-shell effects are measurable or not. It is therefore essential to extend the discussion to the (un)measurability of off-shell effects as well.

For decades the off-shell properties of two-body amplitudes seemed to be a legitimate measurable quantity, and numerous attempts to get hold of it in nucleon-nucleon bremsstrahlung and real and virtual Compton scattering on the nucleon have been made. However, in early 2000s it became apparent that strong field-theoretical arguments do not speak in favor of this claim [2–4]. It seems that it is very likely that the well-known invariance of the scattering matrix with respect to the field transformation of the effective Lagrangian [5] makes it possible to transform the off-shell effects into the contact terms for diagrams of the same power counting level. This effectively makes the off-shell effects an unmeasurable quantity.

When applied to the effective two-body meson-nucleon amplitudes, this statement implies that the ability of coupled-channel formalisms to separate the self-energy term and evaluate the bare scattering matrix poles (singularities in which the meson-exchange effects are fully taken into account) is a model-dependent procedure. Namely, any method for evaluating self-energy contributions unavoidably demands a definite assumption on the analytic form of the off-shell interaction terms, hence introduces model-dependent and consequently unmeasurable hadronic shifts.

The invariance of different parameterizations of the scattering matrix singularities with respect to field redefinitions is also the object of our study. Scattering matrix poles are nowadays quantified in two dominant ways: either as Breit-Wigner parameters, *i.e.* parameters of a Breit-Wigner function which is used to locally represent the experimentally obtainable T-matrix, or as scattering matrix poles (either T or K). In spite of the fact that it is since Hoehler's analysis [6,7] quite commonly accepted that Breit-Wigner parameters are necessarily model-dependent quantities, they are still widely used to quantify the scattering matrix poles. Only recently the scattering matrix poles are being shown in addition. We demonstrate that within the framework of effective field theory, scattering matrix poles are, contrary to Breit-Wigner parameters, unique with respect to arbitrary field redefinitions.

Bare and dressed scattering matrix quantities have for more then a decade been calculated and presented within a framework of various coupled-channel models [8,9], and a definite correlation between scattering matrix singularities and quark-model quantities has been in general established [10]. However, the most direct connection between full scattering matrix singularities and hadron models with confinement forces has been offered in [11–13] for the various versions of dynamical coupled-channel model. In these models the bare N^* states are understood as the excited states of the nucleon if its coupling with the reaction channels is turned off, so the authors naturally speculate that the bare N^* states of these models correspond to the predictions from a hadron model with confinement force, such as the well-developed constituent quark model with gluon-exchange interactions. The role of hadronic shift model dependence, however, is not explicitly discussed.

A simple conclusion emerges: dressed scattering matrix singularities are the best, model-independent meeting point between quark model predictions and experiments, and bare quantities in coupled-channel models remain to be legitimate quantities to be extracted only within a framework of a well-defined model. To understand and interpret them correctly, one has to keep track of the existence of the hadronic mass shifts produced by off-shell ambiguities, and take them fully into account.

2 Dressing and undressing scattering matrix singularities

2.1 Unquenching the quark model

The usual prescription for calculation of the masses of baryons is to ignore the effects of decay-channel couplings, which is the assumption that the states are infinitely long lived. Given that baryon widths are comparable to the mass splittings between similar states caused by short-range interactions between the quarks, the effects on baryon masses of continuum (baryon-meson) states, or equivalently qqq-$q\bar{q}$ components, clearly cannot be ignored. The problem is that there are many distinct intermediate states which can contribute substantially to the self-energies of baryons through baryon-meson loops, because of the presence of many thresholds in the resonance region. Calculations of the effect of two-meson intermediate states in mesons have been carried out, especially for the interesting problem of the ω-ρ mass difference [14,15] which illustrate the complications which arise in the case of baryons. In order to calculate the self-energy of a baryon $B(\mathbf{0})$ due to a particular baryon-meson intermediate state $B'(-\mathbf{k})C(\mathbf{k})$, as in fig. 1, we require a calculation of the dependence of the vertex $\mathcal{M}_{BB'C}(k)$ on the magnitude k of the loop momentum $\mathbf{k}$. This in turn requires a model of the spectrum (including states not seen in experiment), which provides wave functions for the baryons, and a model of the $B(\mathbf{0}) \rightarrow B'(-\mathbf{k})C(\mathbf{k})$ decay vertices. A popular choice for the former is some form of constituent quark model, and for the latter is a pair-creation model such as the $^3P_0$ model illustrated in fig. 2, where baryons decay by the creation of a quark-antiquark pair with the quantum numbers of the vacuum.

In order to self-consistently calculate the masses of baryons in the presence of baryon-meson intermediate states, one possible approach [16] is as follows. The masses and decays are calculated using a three-quark Hamiltonian H_{qqq} and a pair-creation Hamiltonian $H_{\rm pc}$, that depend on strong coupling, quark mass, and string tension parameters α_s^0, m_i^0, and b^0, etc., and a pair-creation coupling

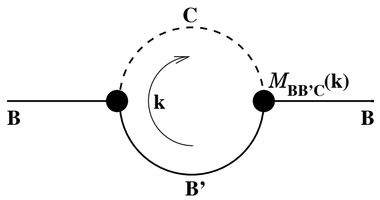

Fig. 1. Calculation of baryon self-energies in the quark model.

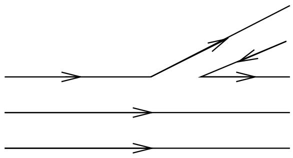

Fig. 2. Pair-creation model of baryon decays.

strength γ. These parameters are usually determined by a fit to the (dressed) spectrum E_B and decay partial widths in the absence of Fock space components higher than qqq,

$$E_B = M_B^0(\alpha_s^0, m_i^0, b^0, \ldots).$$

The correction due to the loop $B \to B'C$ is

$$E_B = M_B^0(\alpha_s^0, m^0, b^0, \ldots) + \Sigma_{B'C}(E_B, E_{B'}; \alpha_s^0, m_i^0, b^0),$$

where

$$\Sigma_{B'C} = \mathcal{P}\int \mathrm{d}^3k \frac{|\langle B'(-\mathbf{k})C(\mathbf{k})|H_{\mathrm{pc}}|B(\mathbf{0})\rangle|^2}{E_B - \sqrt{E_{B'}^2 + k^2} - \sqrt{E_C^2 + k^2} + i\epsilon}.$$

The imaginary part of the loop integral is $\Gamma_{B\to B'C}/2$. A sum is to be performed over baryon-meson intermediate states $B'C$, and the parameters α_s^0, m_i^0, b^0, and $\gamma, \ldots$ are to be adjusted for self-consistent solution with E_B equal to the dressed baryon mass. In principle one should solve similar equations for the meson masses E_C.

This procedure is equivalent to second-order perturbation theory in the decay Hamiltonian H_{pc}, and allows calculation of the (momentum-space) continuum $B'C$ component of the dressed baryon states, and also the mixing $B \to B'C \to B''$ between different baryon states caused by the continuum intermediate states.

Calculation of this kind have been applied to N, Δ, Λ, Σ and Σ^* ground and (singly) orbitally excited states using intermediate states made up of ground-state baryons, with the pseudoscalar mesons π, K, η, η' in ref. [17] and these pseudoscalar mesons plus the vector mesons ρ, ω and K^* in refs. [18–20]. Because there are many baryon-meson thresholds nearby in energy, for example the $N\rho$ and $\Delta\rho$ thresholds are close to those of $N(1535)\pi$ or $\Lambda(1405)K$, one should not restrict the intermediate meson states to π, or even all pseudoscalars, or the intermediate baryon states to N and Δ, or even all octet and decuplet ground states.

Zenczykowski [18] showed that if one assumes exact $SU(3)_f \otimes SU(2)_{\mathrm{spin}}$ symmetry and only ground-state baryons and mesons exist, then all octet and decuplet baryons have the same mass M_B^0 and the same wave function, and also all pseudoscalar and vector ground-state mesons have the same mass M_C^0 and the same wave function, and all self-energy loop integrals are the same, apart from $SU(6)_{\text{spin-flavor}}$ factors at the vertices. Under these conditions we expect the sum of self-energy contributions to the nucleon and $\Delta(1232)$ masses to be identical. Interestingly, the sum of the squares of the $SU(6)_{\text{spin-flavor}}$ factors is the same only if we include all baryon-meson combinations (non-strange, strange, or both) consistent with the conserved quantum numbers, including both pseudoscalar and vector mesons. This is true of the self-energies of any ground-state baryon, and is also true if the $^3P_0$ model is used to calculate the vertex factors, as it reduces to $SU(6)_W$ in this limit.

Away from the $SU(3)_f$ limit, Tornqvist and Zenczykowski [21] were able to show that, with the introduction of a simple pattern of $SU(3)_f$ breaking present in the ground-state baryon and meson mass spectra, the usual $SU(6)$ relations for baryon masses are present in the dressed baryon masses calculated to first order in the symmetry breaking parameters. This suggests that we can interpret $SU(6)$ symmetry breaking effects as partly due to spin- and flavor-dependent interactions between the quarks, and partly due to loop effects.

It is clear from this and other calculations that the effects of these self-energies on the spectrum are substantial. Zenczykowski [18] finds many mass splittings close to those of the dressed pole parameters from analyses, without spin- and flavor-dependent interactions between the quarks. Other calculations show splittings in the dressed P-wave (lowest orbitally) excited baryons which resemble spin-orbit effects [17,19,20]. These could cancel against those expected from other sources and provide a solution to the spin-orbit problem in certain quark models of baryon masses.

These calculations lack a self-consistent treatment of external and intermediate baryon states, and so it is not clear that the sum over intermediate baryon-meson states has converged. Geiger and Isgur [14] demonstrated that this sum does converge, albeit slowly, using a nonrelativistic quark model for baryon masses and wave functions and a $^3P_0$ model for their decays. A covariant model based on the Schwinger-Dyson-Bethe-Salpeter approach was shown to lead to faster convergence in ref. [15]. A study of the dressed masses of N and Δ ground and P-wave excited baryons [22] which involves intermediate pseudoscalar and vector ground-state mesons and many intermediate baryons (ground states, and N, Δ, Λ, Σ, Σ^* excited states up to the second band of negative-parity states at roughly 2100 MeV), representing hundreds of intermediate states, is underway. Vertex form factors are calculated analytically using mixed relativized-model [23] wave functions and the $^3P_0$ model [24].

This study shows that the usual $^3P_0$ model gives vertices which are too hard, giving large contributions from high loop momenta. They can be softened by adopting a pair-creation form factor which decreases as the relative momentum of the created quark and antiquark increases. This calculation is currently being reworked to allow self-consistent renormalization of the quark model parameters.

As an example, in ref. [22] the strong coupling parameter α_s^0 was reduced in order to take into account the additional Δ-N splitting in the sums over baryon-meson loops contributing to the self-energies of both of these states. Similarly, in their calculation of these effects in mesons, Geiger and Isgur [14] showed that the formation of an intermediate meson pair was equivalent to string breaking, which has the effect of renormalizing the meson string tension. Barnes and Swanson [25] have examined shifts in the charmonium spectrum due to D, D^*, D_s and D_s^* meson pairs.

In conclusion, the next Fock space component is likely more important than differences among qqq models. Calculating its effects requires the use of a full set of $SU(6)$-related intermediate states, spatially excited intermediate baryons, and a careful treatment of mixing effects. Renormalization of the parameters in the quark model parameters such as α_s, the quark masses, the string tension, and the pair-creation strength needs to be carried out systematically. This requires examining the mass shifts of more than just N, Δ and their negative-parity excitations. Decay vertices need additional suppression when the dressed masses of external states are well above the threshold for an intermediate state, which is the case in relativistic models.

2.2 Undressing the dressed scattering matrix singularities

To get a better understanding of the relation of bare quantities to dressed quantities it is sufficient to study a system of two nucleon like states (N and R) coupled to a scalar field (σ) [26]. A possible Lagrangian reads

$$\mathcal{L}_1 = \bar{N}\left(i\partial\!\!\!/ - M_N\right)N + \bar{R}\left(i\partial\!\!\!/ - M_R^0\right)R + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m^2\right) + g\sigma\left(\bar{R}N + \bar{N}R\right)\dots \quad (1)$$

Here the superscript 0 indicates that masses are bare quantities that undergo dressing beyond tree level[1]. The resulting vertex is shown as diagram (a) in fig. 3. The dots indicate possible more complex terms, like contact terms of the type $\sigma^2\bar{R}R$ (see fig. 3(c)). However, in phenomenological studies those are rarely included. From this Lagrangian we may now calculate observables like scattering amplitudes. To keep things simple we focus only on the self-energy of the R field. The corresponding diagram is shown in fig. 4(a). The real part of this diagram provides the so-called hadronic shift —the difference between the bare mass and the physical mass— and the imaginary part the width.

A theorem based on very general assumptions in field theory states that *if two fields ϕ and χ are related non-linearly ($\phi = \chi F(\chi)$ with $F(0) = 1$) then the same*

[1] In principle also the coupling g and the mass of σ and N are bare quantities, however, to ease notation we drop the corresponding superscript, for in what follows we focus solely on the self-energy of the R field.

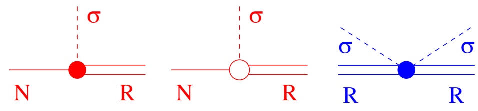

Fig. 3. Vertices from the interaction Lagrangians of eqs. (1) and (3).

observables arise if one calculates with ϕ using $\mathcal{L}(\phi)$ or with χ using $\mathcal{L}(\chi F(\chi))$ [27]. Thus instead of the fields in eq. (1) we may switch to a modified nucleon field defined through

$$N \longrightarrow N' = N + \alpha\sigma R,$$

where α is an arbitrary, real parameter. Expressed in terms of N' the interaction part of the original Lagrangian now reads

$$\mathcal{L}_{1'}^I = g\sigma\left(\bar{R}N' + \bar{N}'R\right) - \alpha\sigma\left(\bar{R}\left(i\partial\!\!\!/ - M_N\right)N' + \text{h.c.}\right) + \sigma\bar{R}\left[\alpha^2\left(i\partial\!\!\!/ - M_N\right) - 2g\alpha\right]R\sigma + \dots . \quad (2)$$

In addition to the vertex of the previous Lagrangian now two new structures appear: a momentum-dependent $RN\sigma$ vertex (depicted in fig. 3(b)) and a $\sigma^2\bar{R}R$ vertex (depicted in fig. 3(c)). The resulting contributions to the R self-energy from this Lagrangian are shown in fig. 4(a)–(d). The field-theoretic theorem quoted above gives that the total self-energy contribution from the modified Lagrangian is identical to that of the original Lagrangian with the same parameters. Especially, the hadronic shift remains the same and it seems that indeed it is a well-defined quantity. However, the problem is that we do not know the *true* hadronic Lagrangian. Thus, starting from eq. (1) is as justified as starting from the following interaction Lagrangian:

$$\mathcal{L}_2^I = g\sigma\left(\bar{R}N + \bar{N}R\right) - \alpha\sigma\left(\bar{R}\left(i\partial\!\!\!/ - M_N\right)N + \text{h.c.}\right) + \dots . \quad (3)$$

Obviously, the only difference to the previous equation is that the $\sigma^2\bar{R}R$ vertex was abandoned. On the one-loop level thus the only difference compared to the previous expression for the R self-energy is that tadpole diagrams were removed. Since this class of diagrams does not lead to non-analyticities, their effect can always be absorbed into the bare mass and the wave function renormalization of the R field. Therefore, with properly adjusted parameters, the self-energy is the same to one loop between the theory that follows from $\mathcal{L}_1$ and that from $\mathcal{L}_2$.

Is there any way in practise to decide, which one of the two Lagrangians is to be preferred? The answer to this question is *no* for the following reasons: although the $\sigma^2\bar{R}R$ contact term can contribute to the R self-energy at three-loop order, this is of no practical significance, since not only has any effective Lagrangian a too limited range of applicability and accuracy to allow for the extraction of such effects but also a complete treatment should include anyway direct $\sigma R \to \sigma R$ transitions in both Lagrangians in addition to the terms given explicitly above. The latter argument also applies to information deduced

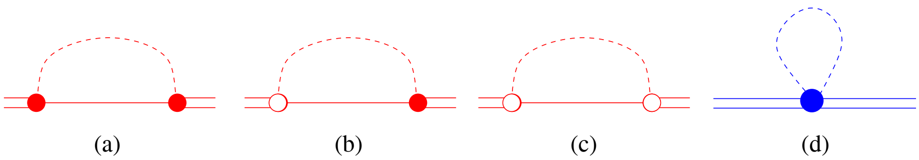

Fig. 4. Self-energies for the R field to one-loop order from toy model I (eq. (1)) and toy model II (eq. (3)).

from $\sigma R \to \sigma R$ cross-sections. Therefore there is in practice no way to decide which one of the two interaction Lagrangians —eq. (1) or eq. (3)— is to be preferred. As outlined above, however, quantities like the self-energies of the resonance R are different in the two approaches and consequently the bare masses as extracted from fits to experiment are different. We therefore conclude that bare masses (or in general bare quantities) do not have any physical significance.

The question studied here is very closely linked to the question of measurability of off-shell effects. The argument just presented can also be used as yet another illustration that off-shell effects are not observable. This already follows from a comparison of eq. (1) and eq. (2). As argued above both lead to identical observables. Especially, the on-shell $RN\sigma$ vertex that can be related to the decay width from $R \to \sigma N$ is the same for both models. However, in our example for off-shell nucleons the vertex can be anything. In general, within a consistent field theory off-shell effects either can be absorbed into counter terms or have to cancel exactly. The same issue is discussed for bremsstrahlung in ref. [4]. Another illustrative example of the cancellation of off-shell effects is provided in ref. [28] for the reaction $NN \to NN\pi$.

It should be stressed that the question in focus here is very different to that of the relation between two-nucleon and three-nucleon observables and the presence of three-body forces. The main difference is that in the few nucleon systems it is possible to construct three-body forces that are consistent with the two-nucleon interaction used, *e.g.* within effective field theory —for a recent review see ref. [29]. Changing the two-nucleon interactions leads also to controlled changes in the three-body forces in the sense sketched above. However, what would be needed for a model-independent extraction of bare hadron masses would be a method to identify *the* hadronic interaction that is the one that matches to the particular quark model, thus a connection is needed between two systems with very different degrees of freedom. We argue that it follows from the reasoning above that this identification cannot be made as a matter of principle. However, the inclusion of hadronic loops within the quark model, as sketched, *e.g.* in the presentation by Simon Capstick, is obviously justified.

We therefore have to conclude that the only quantities relevant for spectroscopy that can be extracted from experiment are resonance poles and the corresponding residues. However, this is still a lot for both quantities contain important structure information like the amount of $SU(3)$ violation or even the very nature of the state [30]. An extraction of poles and residues from the data needs coupled-channel codes of the type of refs. [31–33] with the correct analytical properties and consistent with unitarity. Only then a controlled analytic continuation to the complex plain is possible.

3 Field theory considerations

3.1 From off-shell to on-shell kinematics

It is a natural and legitimate question to ask whether the off-shell behavior of particular interaction vertices is unique and whether it is possible to extract such behavior from empirical information similarly as one, say, extracts the electromagnetic form factors of the nucleon from elastic electron scattering. In this context one might think of the electromagnetic interaction of a bound (off-shell) nucleon or the investigation of the off-shell nucleon-nucleon amplitude entering the nucleon-nucleon bremsstrahlung process. For the case of pions, Compton scattering [34] and pion-pion bremsstrahlung [2] were discussed using chiral perturbation theory (ChPT) at lowest order. It was shown that off-shell effects with respect to the effective pion fields depend on both the model used and the choice of representation for the fields. From that the conclusion was drawn that off-shell effects are not only model dependent but also representation dependent, making a unique extraction of off-shell effects impossible. The spin-1/2 case was discussed in ref. [3].

A related situation occurs when one is interested in corrections to current-algebra results obtained from the partially conserved axial-vector current (PCAC) relation

$$\partial_\mu A^{\mu,a} = M_\pi^2 F_\pi \Phi^a, \tag{4}$$

where $A^{\mu,a}$ is the isovector axial-vector current and Φ^a is a renormalized field operator creating and destroying pions; M_π and F_π denote the pion mass and decay constant, respectively. While predictions of current algebra and the PCAC relation involve the so-called soft-pion limit, $\lim_{q_0\to 0}\lim_{\boldsymbol{q}\to 0}[\cdots]$, amplitudes for physical pions are to be taken at $q^2 = M_\pi^2$. Can the connection between soft-pion kinematics and on-shell kinematics be uniquely determined? The answer is yes, if the problem is entirely formulated in terms of the relevant QCD Green functions.

We will illustrate these issues in the framework of ChPT [35] which establishes a systematic connection with the underlying field theory, namely, QCD. Let us first discuss off-shell effects with respect to the effective fields. To that end, we consider $\pi\pi$ scattering at lowest order in ChPT (see sect. 4.6.2 of ref. [36] for more details):

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}[\partial_\mu U(\partial^\mu U)^\dagger] + \frac{F_\pi^2 M_\pi^2}{4}\,\mathrm{Tr}(U+U^\dagger),$$

where $M_\pi^2 = 2B\hat{m}$. B is related to the quark condensate $\langle\bar{q}q\rangle_0$ in the chiral limit and $\hat{m}$ is the average of the u- and d-quark masses [35]; U is an $SU(2)$-matrix containing the pion fields. We will use two alternative parameterizations of U:

$$U(x) = \frac{1}{F_\pi}\left[\sigma(x) + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\right], \quad \sigma(x) = \sqrt{F_\pi^2 - \boldsymbol{\pi}^2(x)},$$
$$U(x) = \exp\left[i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\phi}(x)}{F_\pi}\right] = \cos\left(\frac{\phi}{F_\pi}\right) + i\boldsymbol{\tau}\cdot\hat{\phi}\sin\left(\frac{\phi}{F_\pi}\right).$$

The σ and exponential parameterizations are related by a field transformation (change of variables)

$$\frac{\boldsymbol{\pi}}{F_\pi} = \hat{\phi}\sin\left(\frac{\phi}{F_\pi}\right) = \frac{\boldsymbol{\phi}}{F_\pi}\left(1 - \frac{1}{6}\frac{\boldsymbol{\phi}^2}{F_\pi^2} + \ldots\right).$$

The relevant $\pi\pi$ interaction Lagrangians read

$$\mathcal{L}_2^{4\pi} = \frac{1}{2F_\pi^2}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi} - \frac{M_\pi^2}{8F_\pi^2}(\boldsymbol{\pi}^2)^2,$$
$$\mathcal{L}_2^{4\phi} = \frac{1}{6F_\pi^2}(\partial_\mu\boldsymbol{\phi}\cdot\boldsymbol{\phi}\partial^\mu\boldsymbol{\phi}\cdot\boldsymbol{\phi} - \boldsymbol{\phi}^2\partial_\mu\boldsymbol{\phi}\cdot\partial^\mu\boldsymbol{\phi}) + \frac{M_\pi^2}{24F_\pi^2}(\boldsymbol{\phi}^2)^2.$$

Observe that the two interaction Lagrangians depend differently on the respective pion fields. For Cartesian isospin indices a, b, c, d the Feynman rules for the scattering process $\pi^a(p_a) + \pi^b(p_b) \to \pi^c(p_c) + \pi^d(p_d)$ read, respectively,

$$\mathcal{M}_2^{4\pi} = i\left(\delta^{ab}\delta^{cd}\frac{s-M_\pi^2}{F_\pi^2} + \delta^{ac}\delta^{bd}\frac{t-M_\pi^2}{F_\pi^2} + \delta^{ad}\delta^{bc}\frac{u-M_\pi^2}{F_\pi^2}\right),$$
$$\mathcal{M}_2^{4\phi} = \mathcal{M}_2^{4\pi} - \frac{i}{3F_\pi^2}\left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right)\left(\Lambda_a + \Lambda_b + \Lambda_c + \Lambda_d\right),$$

where we introduced $\Lambda_k = p_k^2 - M_\pi^2$ and the usual Mandelstam variables $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$, and $u = (p_a - p_d)^2$ satisfying $s + t + u = p_a^2 + p_b^2 + p_c^2 + p_d^2$. If the initial and final pions are all on the mass shell, *i.e.*, $\Lambda_k = 0$, the scattering amplitudes are the same, in agreement with the equivalence theorem of field theory [37]. On the other hand, if one of the momenta of the external lines is off mass shell, the amplitudes $\mathcal{M}_2^{4\pi}$ and $\mathcal{M}_2^{4\phi}$ differ. In other words, a "direct" calculation of $\pi\pi$ scattering in terms of the effective fields gives a unique result independent of the parameterization of U only for the on-shell matrix element.

According to the standard argument in nucleon-nucleon bremsstrahlung one would now try to discriminate between different on-shell equivalent $\pi\pi$ amplitudes through an investigation of the reaction $\pi^a(p_a) + \pi^b(p_b) \to \pi^c(p_c) + \pi^d(p_d) + \gamma(k)$. This claim was critically examined in refs. [2,3]. To that end the electromagnetic field is included through the covariant derivative $D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$, where $Q = \mathrm{diag}(2/3, -1/3)$ is the quark charge matrix. In the σ parameterization, the total bremsstrahlung amplitude is given by the sum of only such diagrams, where the photon is radiated off the initial and final charged pions, respectively. One may then ask how the different off-shell behavior of the $\pi\pi$ amplitude of $\mathcal{M}_2^{4\phi}$ enters into the calculation of the bremsstrahlung amplitude. Observe, in this context, that the exponential parameterization generates electromagnetic interactions involving $2n$ pion fields, where n is a positive integer. In the exponential parameterization an additional $4\phi\gamma$ interaction term relevant to the bremsstrahlung process is generated. Hence the total tree level amplitude now contains an additional four-pion-one-photon contact diagram. Combining the contribution due to the off-shell behavior in the $\pi\pi$ amplitude $\mathcal{M}_2^{4\phi}$ with the contact-term contribution, we found a precise cancelation of off-shell effects and contact interaction such that the final results are the same for both parameterizations. This is once again a manifestation of the equivalence theorem [37]. What is even more important in the present context is the observation that the two mechanisms, *i.e.* contact term *vs.* off-shell effects, are indistinguishable since they lead to the same measurable amplitude.

Now, what about the off-shell behavior of QCD Green functions? The method developed by Gasser and Leutwyler [35] deals with Green functions of color-neutral, Hermitian quadratic forms involving the light-quark fields $q = (u, d)^T$ of QCD and their interrelations as expressed in the Ward identities. In particular, these Green functions can, in principle, be calculated for any value of squared momenta even though ChPT is set up only for a low-energy description. For the discussion of $\pi\pi$ scattering one considers the four-point function [35]

$$G_{PPPP}^{abcd}(x_a, x_b, x_c, x_d) \equiv \langle 0|T[P^a(x_a)\cdots P^d(x_d)]|0\rangle \quad (5)$$

with the pseudoscalar quark density $P^a = i\bar{q}\gamma_5\tau^a q$. In order so see that eq. (5) can indeed be related to $\pi\pi$ scattering, we investigate the matrix element of P^a evaluated between a single-pion state and the vacuum [35]:

$$\langle 0|P^a(0)|\pi^b(q)\rangle \equiv \delta^{ab}G_\pi. \quad (6)$$

The coupling of an external pseudoscalar source p to the Goldstone bosons is given by

$$\mathcal{L}_{\mathrm{ext}} = i\frac{F_\pi^2 B}{2}\mathrm{Tr}(pU^\dagger - Up) = \begin{cases} 2BF_\pi p^a\pi^a, \\ 2BF\pi p^a\phi^a[1 - \boldsymbol{\phi}^2/(6F_\pi^2) + \ldots], \end{cases} \quad (7)$$

where the first and second lines refer to the σ and exponential parameterizations of U, respectively. From eq. (7) we obtain $G_\pi = 2BF_\pi$ independent of the parameterization used which, since the pion is on-shell, is a consequence of the equivalence theorem [37]. As a consistency check, let us verify the PCAC relation from the QCD Lagrangian

$$\partial_\mu A^{\mu,a} = \hat{m}i\bar{q}\gamma_5\tau^a q \equiv \hat{m}P^a,$$

evaluated between a single-pion state and the vacuum. The axial-vector current matrix element obtained from $\mathcal{L}_2$ reads

$$\langle 0|A^{\mu,a}(x)|\pi^b(q)\rangle = iq^\mu F_\pi e^{-iq\cdot x}\delta^{ab}. \quad (8)$$

Taking the divergence implies $M_\pi^2 F_\pi = \hat{m} G_\pi$. In other words,

$$\Phi^a(x) \equiv \frac{\hat{m} P^a(x)}{M_\pi^2 F_\pi} \tag{9}$$

can serve as a so-called *interpolating* pion field in the LSZ reduction formula. Using eq. (9), the reduction formula relates the S-matrix element of $\pi\pi$ scattering to the QCD Green function involving four pseudoscalar densities

$$S_{fi} = \left(\frac{-i}{G_\pi}\right)^4 (p_a^2 - M_\pi^2) \cdots (p_d^2 - M_\pi^2)$$
$$\times \int \mathrm{d}^4 x_a \cdots \mathrm{d}^4 x_d \, e^{-ip_a\cdot x_a} \cdots e^{ip_d \cdot x_d} G^{abcd}_{PPPP}(x_a, x_b, x_c, x_d).$$

Using translational invariance, let us define the momentum space Green function as

$$(2\pi)^4 \delta^4(p_a + p_b + p_c + p_d) F^{abcd}_{PPPP}(p_a, p_b, p_c, p_d) =$$
$$\int \mathrm{d}^4 x_a d^4 x_b d^4 x_c d^4 x_d \, e^{-ip_a\cdot x_a} e^{-ip_b\cdot x_b} e^{-ip_c\cdot x_c} e^{-ip_d x_d}$$
$$\times G^{abcd}_{PPPP}(x_a, x_b, x_c, x_d),$$

where we define all momenta as incoming. The usual relation between the S-matrix and the T-matrix, $S = I + iT$, implies for the T-matrix element $\langle f|T|i\rangle = (2\pi)^4\delta^4(P_f - P_i)\mathcal{T}_{fi}$ and, finally, for $\mathcal{M} = i\mathcal{T}_{fi}$:

$$\mathcal{M} = \frac{1}{G_\pi^4}\left[\prod_{k=a,b,c,d} \lim_{p_k^2 \to M_\pi^2} (p_k^2 - M_\pi^2)\right] F^{abcd}_{PPPP} \tag{10}$$

with $F^{abcd}_{PPPP} \equiv F^{abcd}_{PPPP}(p_a, p_b, -p_c, -p_d)$. We will now determine the Green function F^{abcd}_{PPPP} using the σ and exponential parameterizations for U. In the first parameterization we only obtain a linear coupling between the external pseudoscalar field and the pion field (see eq. (7)) so that only one Feynman diagram contributes

$$F^{abcd}_{PPPP} = (2BF_\pi)^4 \frac{i}{p_a^2 - M_\pi^2} \cdots \frac{i}{p_d^2 - M_\pi^2} \mathcal{M}_2^{4\pi}. \tag{11}$$

The Green function F^{abcd}_{PPPP} depends on six independent Lorentz scalars which can be chosen as the squared invariant momenta p_k^2 and the three Mandelstam variables s, t, and u satisfying the constraint $s + t + u = \sum_k p_k^2$.

Using the second parameterization we will obtain a contribution which is of the same form as eq. (11) but with $\mathcal{M}_2^{4\pi}$ replaced by $\mathcal{M}_2^{4\phi}$. Clearly, this is not yet the same result as eq. (11) because of the terms proportional to Λ_k in $\mathcal{M}_2^{4\phi}$. However, in this parameterization the external pseudoscalar field also couples to three pion fields (see eq. (7)), resulting in four additional contributions

$$\Delta_a F^{abcd}_{PPPP} + \cdots + \Delta_d F^{abcd}_{PPPP}$$

with

$$\Delta_a F^{abcd}_{PPPP}(p_a, p_b, -p_c, -p_d) =$$
$$(2BF_\pi)^4 \frac{i}{p_a^2 - M_\pi^2} \cdots \frac{i}{p_d^2 - M_\pi^2}$$
$$\times \frac{i\Lambda_a}{3F_\pi^2}(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}), \tag{12}$$

and analogous expressions for the remaining ΔF's. In total, we find a complete cancelation with those terms proportional to Λ_k (in the second parameterization) and the end result is identical with eq. (11)! Finally, using $G_\pi = 2BF_\pi$ and inserting the result of eq. (11) into eq. (10) we obtain the same scattering amplitude as in the "direct" calculation of $\mathcal{M}_2^{4\pi}$ and $\mathcal{M}_2^{4\phi}$ evaluated for on-shell pions.

This example serves as an illustration that the method of Gasser and Leutwyler generates unique results for the Green functions of QCD for arbitrary four-momenta. There is no ambiguity resulting from the choice of variables used to parameterize the matrix U in the effective Lagrangian. These Green functions can be evaluated for arbitrary (but small) four-momenta. Using the reduction formalism, on-shell matrix elements such as the $\pi\pi$ scattering amplitude can be calculated from the QCD Green functions. The result for the $\pi\pi$ scattering amplitude as derived from eq. (10) agrees with the "direct" calculation of the on-shell matrix elements of $\mathcal{M}_2^{4\pi}$ and $\mathcal{M}_2^{4\phi}$. On the other hand, the Feynman rules of $\mathcal{M}_2^{4\pi}$ and $\mathcal{M}_2^{4\phi}$ when taken *off-shell*, have to be considered as intermediate building blocks only and thus need not be unique.

3.2 Model (in)dependence of pole positions and Breit-Wigner parameters

A popular definition of masses of unstable particles corresponding to a (relativistic) Breit-Wigner formula makes use of the zero of the real part of the inverse propagator. It has been shown that such a definition leads to field redefinition and gauge-parameter dependence of the mass starting at two-loop order [38–44]. In contrast, defining the mass and width in terms of the complex-valued position of the pole of the propagator leads to both field redefinition and gauge-parameter independence.

As the baryon resonances are thought to be described by QCD, with the progress of lattice techniques and, especially, the low-energy effective theories (EFT) of QCD (see, *e.g.*, [45,35,46–50] and references therein) the question of defining baryon resonance masses becomes important. Here we examine this issue for the Δ-resonance. As discussed in ref. [7], the *conventional resonance mass* and width determined from generalized Breit-Wigner formulas have problems regarding their relation to the S-matrix theory and suffer from a strong model dependence. Here, we will show that these parameters, in addition, depend on the field redefinition parameter in a low-energy EFT of QCD.

For simplicity we ignore isospin and consider an EFT of a single nucleon, pion, and Δ-resonance. Defining

$$\Lambda_{\mu\nu} = -(i\,\partial\!\!\!/ - m_\Delta)g_{\mu\nu} + i\,(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) - i\gamma_\mu \partial\!\!\!/ \gamma_\nu - m_\Delta \gamma_\mu\gamma_\nu,$$

the free Lagrangian is given by

$$\mathcal{L}_0 = \bar{\psi}^\mu \Lambda_{\mu\nu}\psi^\nu + \bar{\Psi}(i\,\partial\!\!\!/ - m_N)\Psi + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi. \tag{13}$$

Here, the vector-spinor ψ^μ describes the Δ in the Rarita-Schwinger formalism [51], Ψ stands for the nucleon field with mass m_N, and π represents the pion field which we take massless to simplify the calculations. The interaction terms have the form

$$\mathcal{L}_{\rm int} = g\,\partial^\nu \pi\,\bar{\psi}^\mu\,(g_{\mu\nu} - \gamma_\mu\gamma_\nu)\,\Psi + {\rm H.c.} + \dots\,, \qquad (14)$$

where the ellipsis refers to an infinite number of interaction terms which are present in the EFT. These terms also include all counter terms which take care of divergences appearing in our calculations. Although our results are renormalization scheme independent, for simplicity we use the dimensional regularization with the minimal subtraction scheme.

Let us consider the field transformation

$$\bar{\psi}^\mu \to \bar{\psi}^\mu + \xi\,\partial^\mu\pi\bar{\Psi}, \qquad \psi^\nu \to \psi^\nu + \xi\,\partial^\nu\pi\Psi, \qquad (15)$$

where ξ is an arbitrary real parameter. When inserted into the Lagrangians of eqs. (13) and (14), the field redefinition generates additional interaction terms. The terms linear in ξ are given by

$$\mathcal{L}_{\rm add\,int} = \xi\,\partial^\mu\pi\,\bar{\Psi}\,\Lambda_{\mu\nu}\,\psi^\nu + \xi\,\partial^\nu\pi\,\bar{\psi}^\mu\,\Lambda_{\mu\nu}\,\Psi. \qquad (16)$$

Because of the equivalence theorem physical quantities cannot depend on the field redefinition parameter ξ. The complex-valued position of the pole of the Δ propagator does not depend on ξ. In contrast, the mass and width defined via (the zero of) the real and imaginary parts of the inverse propagator depend on ξ at two-loop order.

The dressed propagator of the Δ in n space-time dimensions can be written as

$$-\,i\left[g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{n-1} - \frac{p^\mu\gamma^\nu - \gamma^\mu p^\nu}{(n-1)m_\Delta} - \frac{(n-2)p^\mu p^\nu}{(n-1)m_\Delta^2}\right] \times \frac{1}{\not p - m_\Delta - \Sigma_1 - \not p\,\Sigma_6} + \text{pole-free terms}, \qquad (17)$$

where we parameterize the self-energy of the Δ as

$$\Sigma_1(p^2)g^{\mu\nu} + \Sigma_2(p^2)\gamma^\mu\gamma^\nu + \Sigma_3(p^2)p^\mu\gamma^\nu + \Sigma_4(p^2)\gamma^\mu p^\nu + \Sigma_5(p^2)\,p^\mu p^\nu + \Sigma_6(p^2)\,\not p\,g^{\mu\nu} + \Sigma_7(p^2)\,\not p\,\gamma^\mu\gamma^\nu + \Sigma_8(p^2)\,\not p\,p^\mu\gamma^\nu + \Sigma_9(p^2)\,\not p\,\gamma^\mu p^\nu + \Sigma_{10}(p^2)\,\not p\,p^\mu p^\nu. \qquad (18)$$

The complex pole z of the Δ propagator is obtained by solving the equation

$$z - m_\Delta - \Sigma_1(z^2) - z\,\Sigma_6(z^2) = 0. \qquad (19)$$

The pole mass is defined as the real part of z.

On the other hand, the mass m_R and width Γ of the Δ-resonance are often determined from the real and imaginary parts of the inverse propagator (corresponding to the Breit-Wigner parametrization), *i.e.*,

$$m_R - m_\Delta - {\rm Re}\,\Sigma_1(m_R^2) - m_R\,{\rm Re}\,\Sigma_6(m_R^2) = 0,$$
$$\Gamma = -2\,{\rm Im}\,\Sigma_1(m_R^2) - 2\,m_R\,{\rm Im}\,\Sigma_6(m_R^2). \qquad (20)$$

We have calculated the Δ mass using both definitions and analyzed their ξ-dependence to first order (for details see ref. [52]).

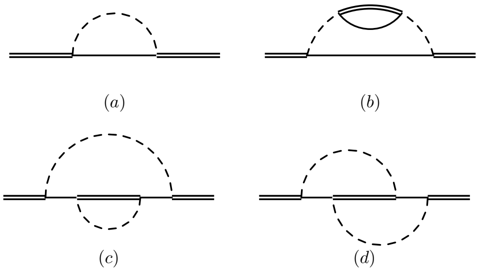

Fig. 5. Δ self-energy diagrams. Solid, dashed, and double lines correspond to nucleon, pion, and Δ, respectively.

The Δ self-energy at one loop-order is given by the diagram in fig. 5(a). The two-loop contributions to the Δ self-energy are given in fig. 5(b)–(d). We are interested in terms linear in ξ.

To find the pole of the propagator we insert its loop expansion

$$z = m_\Delta + \delta_1 z + \delta_2 z + \dots \qquad (21)$$

in eq. (19) and solve the resulting equation order by order.

The one-loop diagram results in the ξ-independent expression for $\delta_1 z$. Calculating diagram (b) and (c) we find that they give vanishing contributions. The ξ-dependent contributions in $\delta_2 z$, generated by the one-loop diagram and by diagram (d) exactly cancel each other leading to the ξ-independent pole of the propagator.

We perform the same analysis inserting the loop expansion of m_R,

$$m_R = m_\Delta + \delta_1 m + \delta_2 m + \dots, \qquad (22)$$

in eq. (20). For δm_1 the one-loop diagram gives a ξ-independent expression. On the contrary, the ξ-dependent contributions in $\delta_2 m$, generated by the one-loop diagram and by diagram (d) do not cancel, thus leading to a ξ-dependent mass m_R. An analogous result holds for the width Γ obtained from eq. (20).

To conclude, we addressed the issue of defining the mass and width of the Δ-resonance in the framework of a low-energy EFT of QCD. In general, the scattering amplitude of a resonant channel can be presented as a sum of the resonant contribution expressed in terms of the dressed propagator of the resonance and the background contribution. The resonant contribution defines the Breit-Wigner parameters through the real and imaginary parts of the inverse (dressed) propagator. The resonant part and the background separately depend on the chosen field variables, while the sum is of course independent of this choice. We have performed a particular field transformation with an arbitrary parameter ξ in the effective Lagrangian. In a two-loop calculation we have demonstrated that the mass and width of the Δ-resonance determined from the real and imaginary parts of the inverse propagator depend on the choice of field variables. On the other hand, the complex pole of the full propagator does not depend on the field transformation parameter ξ.

The above conclusions are not restricted to the considered toy model or EFT in general. Rather, our results are a manifestation of the general feature that the (relativistic) Breit-Wigner parametrization leads to model- and process-dependent masses and widths of resonances. Although in some cases (like the Δ-resonance) the background has small numerical effect on the Breit-Wigner mass, still the pole mass and the width should be considered preferable as these are free of conceptual ambiguities.

4 Bare and dressed quantities within a well-defined model

4.1 Longitudinal and transverse helicity amplitudes of nucleon resonances in a constituent quark model —bare vs. dressed resonance couplings

Many models have been built and applied to the description of hadron properties. An important role is played by Constituent Quark Models (CQM), in which quarks are considered as effective degrees of freedom. There are many versions of CQM, which differ according to the chosen quark dynamics: h.o. and three-body force [53], algebraic [54], hypercentral [55], Goldstone boson exchange [56], instanton [57]. Here we report some results of the hypercentral CQM (hCQM) [55] on the longitudinal and transverse helicity amplitudes of the nucleon resonances. In this model, the quark interaction is assumed to be given by a hypercentral potential

$$V(x) = -\tau/x + \alpha x, \qquad x = \sqrt{\rho^2 + \lambda^2}, \qquad (23)$$

where x is the hyperradius expressed in terms of the internal Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$. A Coulomb-like plus linear confinement form of the potential is supported by recent lattice QCD evaluations of the quark-antiquark potential [58] and in this sense eq. (23) can be considered as the hypercentral approximation of a two-body Cornell-like potential. The model interaction is completed by adding a standard spin-dependent hyperfine interaction H_{hyp} [53], in order to reproduce the splittings within the $SU(6)$ multiplets. The few free parameters (α, τ and the strength of H_{hyp}) are fitted to the spectrum and the model is then applied to calculate (*i.e.*, to predict) various properties of hadrons: the photocouplings [59], the transverse helicity amplitudes for negative-parity resonances [60], the elastic form factors [61], the longitudinal and transverse helicity amplitudes of all the main resonances [62].

It is interesting to analyze in a systematic way the Q^2 behaviour of the helicity amplitudes in comparison with the existing data. Figure 6 shows the results for the transverse helicity amplitudes of the F_{15}-resonance, results which are typical for a $J > 1/2$ state [60,62]: the medium-high Q^2 behaviour is quite well reproduced, showing that the hyper-Coulomb part of the interaction $1/x$ gives a fair account of the short range, while at low Q^2 there is a lack of strength, particularly for the $A_{3/2}$ amplitude. For $J = 1/2$ states [59,62], there are some minor problems in the low-Q^2 region, but for the rest the agreement with data is satisfactory. Major problems are

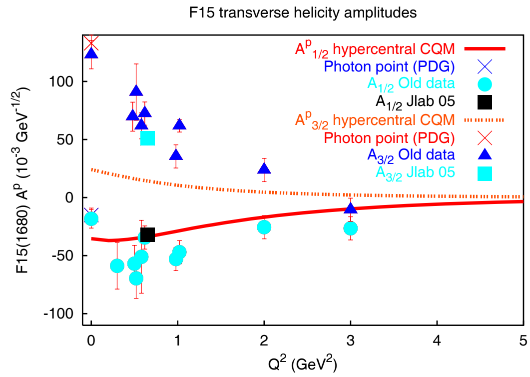

Fig. 6. The transverse helicity amplitudes for the F_{15}-resonance obtained with the hCQM [62], compared with the experimental data, taken by an old compilation [63], recent JLab experiments [64] and PDG [65].

present for the Roper resonance [62], a fact that may support the idea of a particular status of the radial excitations of the nucleon. Discrepancies are present also for the Δ-resonance [62], a feature which is typical of all CQMs; it is well known that the quark model, while reproducing quite well the baryon magnetic moments, fails in the case of the N-Δ transition magnetic moment. Taking into account the fact that the proton radius, calculated with the wave functions corresponding to the potential of eq. (23), turns out to be about 0.5 fm, the emerging picture is that of a small quark core surrounded by an external region, which is probably dominated by dynamical effects not present in the CQM, that is sea-quark or meson cloud effects [60].

These considerations are relevant in connection with the issue of bare *vs.* dressed quantities. One should not forget that the separation between bare and dressed quantities is meaningful within a definite theoretical approach. In CQM calculations the aim is not a fit but the description of observables, which in principle are dressed quantities (like baryon masses, magnetic moments, helicity amplitudes, etc.). In any case the identification of quark results with bare quantities is questionable in view of the fact that CQs have a mass and some dressing is implicitly taken into account. However, a consistent and systematic CQM approach may be helpful in order to put in evidence explicit dressing effects.

These effects have been recently calculated by means of a dynamical model [66]. The meson cloud contribution to various helicity amplitudes has been calculated and compared with the hCQM predictions [67]. The two contributions cannot be added, since they are calculated within different frameworks, however it is interesting to note that the meson cloud contribution is relevant at low Q^2 and in most cases it is important where the hCQM prediction underestimates the data. An example of this situation is given by the longitudinal and transverse helicity amplitudes for the Δ excitation [67]. The case of the $S_{1/2}$ amplitude is particularly interesting (see fig. 7): the hCQM is almost vanishing and the meson cloud contribution accounts for practically the whole strength.

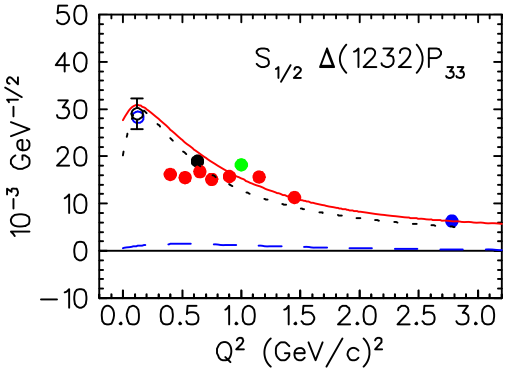

Fig. 7. The Q^2-dependence of the $N \to \Delta$ longitudinal helicity amplitude $S_{1/2}$: superglobal fit performed with MAID [68] (solid curve), predictions of the hypercentral constituent quark model [55,67,62] (dashed curve), pion cloud contributions calculated with the Mainz dynamical model [66] (dotted curve). The data points at finite Q^2 are the results of single-Q^2 fits [67] on recent data quoted in ref. [67].

The problem is how to introduce dressing in the calculations. One way is to adopt a hadronic approach: meson and baryons (nucleon and nucleon resonances) are the relevant degrees of freedom and the dynamics is given by meson-baryon interactions. This is certainly a consistent approach which has been used with success by various groups with different techniques (see, *e.g.*, [68–70]). Another possibility is given by the so-called hybrid models, where the baryons are considered as three-quark states surrounded by a pion cloud and a direct quark-meson coupling is introduced. In this way the electromagnetic excitation acquires contributions also from the meson cloud. This approach is very useful for preliminary calculations (see ref. [71]), however a more promising method is provided by the inclusion of dressing mechanisms directly at the quark level. This means in particular the inclusion of higher Fock components in the baryon state:

$$|\Psi_B\rangle = \Psi_{3q}\ |qqq\rangle + \Psi_{3q\ q\bar{q}}\ |qqq\ q\bar{q}\rangle \qquad (24)$$

and implies the necessity of unquenching the quark model, as discussed in sect. 2.1. For the case of mesons, there are pioneering works by Geiger and Isgur [72], where the $q\bar{q}$ pair creation mechanism is introduced at the microscopic level within a string model. In the case of baryons, the problem is more complicated and has been recently treated performing the sum over intermediate quark loops by means of group theoretic methods [73]. This approach has been applied to the determination of the strange content of the proton [74] with good results. In this way we have at our disposal a promising method for obtaining an unquenched, that is dressed, formulation of the CQM. The systematic calculation of baryon properties, such as transition amplitudes (but also elastic form factors and structure functions) in an unquenched CQM will supply a set of dressed quantities to be compared directly with data and will allow to understand where meson cloud or (better) $q\bar{q}$ effects are important.

One should, however, be aware of some problems, both phenomenological and theoretical. From the phenomenological point of view, there is the problem of the sign of the helicity amplitudes, which is actually extracted from the meson electroproduction amplitude, the fact that the PDG photon points are often non-consistent and the need of new and systematic data. The main theoretical problems are connected with the inclusion of relativity. The kinematic relativistic corrections seem to be not important for the helicity amplitudes [75], however, relativity should be consistently included both in the (unquenched) CQM states and in the transition operators, leading to the possibility of quark pair terms in the electromagnetic current. In any case, the unquenching of the CQM is expected to produce a substantial improvement in the theoretical description of baryon properties. In particular, it will be possible to calculate simultaneously the electromagnetic processes and the strong decays and the baryons resonances will acquire a non-zero width through the coupling to the continuum part of the spectrum.

4.2 Nucleon resonances and hadron structure calculations

4.2.1 What are the nucleon resonances?

To answer this question, it is useful to recall some textbook (for example, see refs. [76–78]) definition of resonances. Phenomenologically, a resonance (R) is identified with a peak in a plot of the reaction cross-section as a function of the collision energy E or invariant mass W. At energies near the peak position, one can fit such data *near* the peak by

$$\sigma_{a,b}(W \sim m_R) \sim \rho(W)\frac{|\Gamma_{R,a}|^2|\Gamma_{R,b}|^2}{(W-m_R)^2+|\frac{\Gamma_0}{2}|^2}, \qquad (25)$$

where $\rho(W)$ is an appropriate phase space factor, m_R the position of the peak, and Γ_0 the width of the peak. The expression eq. (25) has the same function form of the decay rate of an unstable system with a mass $\sim m_R$ and a lifetime $\tau_R \sim 1/\Gamma_0$. It is thus natural to interpret that the cross-section eq. (25) is due to the excitation of an unstable system during the collision. How this unstable system is formed from the entrance and exit channels is a dynamical question which can only be answered by modeling the reaction mechanisms and the internal structure of all particles involved. We will address this question within the Hamiltonian formulation of the problem. This is rather different from the S-matrix approach.

Within the Hamiltonian formulation, there are two ways to derive the expression eq. (25) depending on the structure of the excited unstable system. Let us first consider the one defined in Feshbach's textbook (p. 23 of ref. [78]). It can be stated as the following:

A resonance is formed in a process that the incident projectile completely lose its identity, amalgamating with the target system to form a compound state. Namely, the *evolution of the whole system cannot be defined in terms of the motion of the projectile and its transmutation.*

The expression eq. (25) corresponding to this definition of resonances can be formulated by assuming that the Hamiltonian of the system has the following form:

$$H = H_0 + H' \tag{26}$$

with

$$H' = \sum_a \Gamma_{R,a}, \tag{27}$$

where $\Gamma_{R,a}$ defines the decay of an unstable system R with a mass M_0 into channel a. The reaction amplitude is defined by

$$T(E) = H' + H' \frac{1}{E - H + i\epsilon} H' \tag{28}$$

From eqs. (27), (28), it is straightforward to see that the reaction cross-section for $b \to a$ can be written as

$$\sigma_{a,b}(W) = \rho(W) |T^R_{a,b}(W)| \tag{29}$$

with

$$T^R_{a,b}(W) = \frac{\Gamma_{R,a}(k_a)\Gamma_{R,b}(k_b)}{W - M_0 - \Sigma(W)}, \tag{30}$$

where k_a is the on-shell momentum of channel a and

$$\Sigma_R(W) = \sum_a \langle R|\Gamma^\dagger_{R,a} \frac{1}{W - H_0 + i\epsilon} \Gamma_{R,a}|R\rangle \tag{31}$$

We can cast eq. (30) into

$$T^R_{a,b}(W) = \frac{\Gamma^*_{R,a}(k_a)\Gamma_{R,b}(k_b)}{W - M_R(W) + i\frac{\Gamma^{tot}(W)}{2}}, \tag{32}$$

where

$$M_R(W) = M^0 + \mathrm{Re}(\Sigma_R(W)), \tag{33}$$

$$\frac{\Gamma^{tot}(W)}{2} = -\mathrm{Im}(\Sigma_R(W)). \tag{34}$$

By using eqs. (29) and (32) to fit the expression eq. (25), the parameters of the Hamiltonian are then related to the data by the following relations:

$$m_R = M_R(W = m_R) = M^0 + \mathrm{Re}(\Sigma_R(m_R)), \tag{35}$$

$$\Gamma_0 = \Gamma^{tot}(W = m_R) = -2\,\mathrm{Im}(\Sigma_R(m_R)). \tag{36}$$

Equations (35), (36) then allow us to use the experimental values m_R and Γ_0 to extract the property of the unstable system, specified by M_0 and $\Gamma_{R,a}$ of the Hamiltonian, through the evaluation of eqs. (31)–(33) at energies near $W = m_R$.

The second mechanism which can also yield a cross-section of the form of eq. (25) is

An unstable system is formed during the collision by an attractive force between the interacting particles which do not lose their identities.

The simplest parameterization of an attractive force is a separable potential

$$H' = g^\dagger \frac{1}{C} g. \tag{37}$$

The solution of eq. (28) then becomes

$$T(W) = \frac{g^*(k)g(k)}{C - z(W)}, \tag{38}$$

where

$$z(W) = \langle g| \frac{1}{W - H_0 + i\epsilon} |g\rangle. \tag{39}$$

If the parameters of H' are chosen such that $C - \mathrm{Re}(z(W)) \to 0$ on the physical world $W \to W_0$, where W_0 is a real number, we can expand

$$\begin{aligned} &C - z(W) = \\ &[C - R(W_0) - R'(W_0)(W - W_0) + \cdots] - iI(W) \\ &\sim -R'(W_0) \\ &\left[W - W_0 - \frac{1}{R'(W_0)}(R(W_0) - C) + i\frac{1}{R'(W_0)} I(W)\right], \end{aligned} \tag{40}$$

where $R(W_0) = \mathrm{Re}(z(W_0))$, $I(W) = \mathrm{Im}(z(W))$, and $R'(W_0) = \partial\,\mathrm{Re}(z(W))/\partial W|_{W=W_0}$. We then can write at $W \to W_0$

$$\begin{aligned} &T(W \sim W_0) = \\ &\frac{-g^*_a(k_0)\frac{1}{R'(W_0)} g_b(k_0)}{W - [W_0 + \frac{1}{R'(W_0)}(R(W_0) - C)] + i\frac{1}{R'(W_0)} I(W_0)}. \end{aligned} \tag{41}$$

The above expression can give the resonant cross-section eq. (25) if the parameters of g and C can be chosen to satisfy

$$m_R = W_0 + \frac{1}{R'(W_0)}(R(W_0) - C), \tag{42}$$

$$\frac{\Gamma_0}{2} = \frac{1}{R'(W_0)} I(W_0). \tag{43}$$

4.2.2 Dynamical models for investigating nucleon resonances

From the above two examples, we see that the resonant cross-section eq. (25) can correspond to two very different internal structures of the excited unstable system. The nucleon resonances we are interested in correspond to the unstable systems defined by the Hamiltonian eq. (27). For the meson-nucleon reactions, such unstable systems are due to the excitation of the quark-gluon substructure of the nucleon.

In reality, the situation is much more complicated. In the reactions involving composite systems, such as atoms, nuclei and nucleons, the excitations of resonances always involve non-resonant direct interactions. For example, the non-resonant interactions in pion-nucleon scattering could be due to the exchange of the ρ-meson. The reaction formulation for analyzing such reactions is well presented in Feshbach's textbook [78]. We now briefly describe how such a formulation can be used to investigate nucleon resonances in meson-nucleon reactions.

The starting point is to divide the Hilbert space into a P-space for the entrance and exit channels and Q for the rest. One can cast the equation of motion in the P-space as

$$(E - H_{eff})P\Psi = 0, \tag{44}$$

where

$$H_{eff} = H_{PP} + H_{PQ}\frac{1}{E^{(+)} - H_{QQ}}H_{QP}\,. \tag{45}$$

Here $E^{(+)} = E + i\epsilon$ specifies the boundary condition and we have defined projected operator $H_{PP} = PHP$, $H_{PQ} = PHQ$ and $H_{QQ} = QHQ$. Now consider the eigenstates of H_{QQ} which can be discrete bound Φ_s or unbound $\Phi_{\epsilon,\alpha}$ states

$$H_{QQ}\Phi_s = \epsilon_s\Phi_s, \tag{46}$$
$$H_{QQ}\Phi(\epsilon,\alpha) = \epsilon\Phi(\epsilon,\alpha) \tag{47}$$

with

$$\langle\Phi_s|\Phi_{s'}\rangle = \delta_{s,s'}, \tag{48}$$
$$\langle\Phi(\epsilon,\alpha)|\Phi(\epsilon',\alpha')\rangle = \delta_{\alpha,\alpha'}\delta(\epsilon-\epsilon'). \tag{49}$$

We then expand

$$H_{eff} - H_{PP} = \sum_s \frac{H_{PQ}|\Phi_s\rangle\langle\Phi_s|H_{QP}}{E-\epsilon_s} + \int \mathrm{d}\alpha \int \mathrm{d}\epsilon \frac{H_{PQ}|\Phi(\epsilon,\alpha)\rangle\langle\Phi(\epsilon,\alpha)|H_{QP}}{E^{(+)}-\epsilon}\,. \tag{50}$$

One can see from the above equation that rapid energy dependence of H_{eff} will occur as the energy approaches one of the bound-state energy ϵ_s. This is the origin of rapid energy dependence of the cross-sections. As shown in Feshbach's book (pp. 158–162), the amplitude at $E \sim \epsilon_s$ can be written as

$$T_{fi} = T^P_{fi} + \frac{\langle\chi^{(-)}|H_{PQ}|\Phi_s\rangle\langle\Phi_s|H_{QP}\chi^{(+)}\rangle}{E-\epsilon_s-\langle\Phi_s|W_{QQ}|\Phi_s\rangle} \tag{51}$$

with

$$W_{QQ} = H_{QP}\frac{1}{E^{(+)}-\hat{H}_{PP}}H_{PQ}, \tag{52}$$

where

$$\hat{H}_{PP} = H_{PP} + \int \mathrm{d}\alpha \int \mathrm{d}\epsilon \frac{H_{PQ}|\Phi(\epsilon,\alpha)\rangle\langle\Phi(\epsilon,\alpha)|H_{QP}}{E^{(+)}-\epsilon} \tag{53}$$

and $\chi^{(\pm)}$ are the solutions of

$$(E - \hat{H}_{PP})\chi^{(+)} = 0, \tag{54}$$
$$(E - \hat{H}_{PP})\chi^{(-)*} = 0. \tag{55}$$

In this formulation, one bound state of H_{QQ} will correspond to one resonance. Namely, one can predict whether a resonance can appear in a particular partition of Hilbert space by examining whether bound states can be generated from the Hamiltonian when the coupling with the states P is turned off.

We now point out that the dynamical model developed in ref. [8] (MSL model) for investigating meson-baryon reactions is completely consistent with the formulation given in eqs. (51)–(55). To see this, one just make the following identifications:

- the P-space contains reaction channels $MB = \pi N$, $\gamma N, \eta N, \pi\Delta, \rho N, \sigma N$ and $\pi\pi N$,

$$P = \sum_{MB}|MB\rangle\langle MB| + |\pi\pi N\rangle\langle\pi\pi N|; \tag{56}$$

- H_{QQ} describes the internal structure of the bare N^* states

$$H_{QQ}|N^*_i\rangle = M^0_{N^*_i}|N^*_i\rangle,$$
$$Q = \sum_i |N^*_i\rangle\langle N^*_i|; \tag{57}$$

- H_{PP} defines the non-resonant meson-baryon interactions

$$H_{PP} = \sum_{MB}|MB\rangle\left[\sqrt{m_B+p^2}+\sqrt{m_B+\boldsymbol{k}^2}\right]\langle MB| + \sum_{MB,M'B'} v_{MB,M'B'} + \sum_{MB}[v_{MB,\pi\pi N}+v_{\pi\pi N,MB}] + v_{\pi\pi N,\pi\pi N}; \tag{58}$$

- H_{QP} defines the coupling of the internal structure of N^* with the reaction channels

$$H_{QP} = \sum_{N^*}\left[\sum_{MB}\Gamma_{N^*,MB} + \Gamma_{N^*,\pi\pi N}\right]. \tag{59}$$

With some inspections, one can see that equations presented in the sect. 3 of ref. [8] (MSL model) are completely equivalent to eqs. (51)–(55). If we set $MB = \pi N, \gamma N$ and $Q = |\Delta\rangle\langle\Delta|$, and neglect the $\pi\pi N$ channel, we then obtain the formulation of the SL model [11].

4.2.3 Relations with hadron structure calculations

We now note that ϵ_s and Φ_s in eqs. (46) and (51) relate the structure calculations for the unstable systems in the Q-space and the reaction amplitudes in the P-space. In the MSL formulation, these are the bare mass $M^0_{N^*_i}$ and wave function $|N^*\rangle$ of the discrete *bound* states defined by eq. (57). These bare N^* states can be considered as the excited states of the nucleon if its coupling with the reaction channels is turned off. It is therefore natural to speculate that the bare N^* states in the SL and MSL models correspond to the predictions from a hadron model with confinement force, such as the well-developed constituent quark model with gluon-exchange interactions.

This was first noticed in the SL model in 1996. The idea was later pursued in ref. [13] in 2000 in an attempt to directly calculate the πN scattering amplitude in S_{11} up to $W = 2\,\text{GeV}$ starting from several constituent quark models. In the later $N(e, e'N)$ analysis [12,79] based on the SL model, the bare $\gamma N \to \Delta(1232)$ form factors were also found to be close to the constituent quark model predictions. To consider constituent quark models with meson-exchange residual interactions, the SL and MSL models must be modified to account for the contribution due to the continuum in the Q-space; namely the effects due to the second term in the right-hand side of eq. (53).

It is unlikely that the Lattice QCD calculation (LQCD) can account for the channel coupling effects and unitarity conditions, which are the essential elements of a dynamical coupled-channel analysis, rigorously in the near future. It is a challenging problem to relate the LQCD calculations to the information which can be extracted from the full solution eq. (51) of a dynamical coupled-channel analysis.

One possibility is to perform a LQCD calculation which defines a H_{QQ} of a dynamical coupled-channel analysis. At the present time, perhaps the predictions from a quenched LQCD with heavy quark mass and *no* chiral extrapolation correspond to the bare parameters resulted from the dynamical coupled-channel analysis being performed at EBAC. This is based on the argument that the quark-loop contributions are suppressed at the heavy quark limit and the LQCD mainly accounts for the gluonic interactions which are not in the P-space of MSL model.

5 Conclusion

This work was motivated by problems relating the reliable results from partial-wave and amplitude analysis, which are the parameters of dressed scattering matrix singularities, and the results of quark models, which are usually given as the properties of bare resonances. Undressing dressed scattering matrix singularities in coupled-channel models involves model-dependent hadronic mass shifts, which arise from the unmeasurability of off-shell effects accompanying the dressing procedure. It is legitimate to extract bare quantities in coupled-channel models within the framework of a well-defined model, but their interpretation requires keeping track of hadronic mass shifts produced by off-shell ambiguities. The best meeting point between quark model predictions and analyses of experimental data are dressed scattering matrix singularities, as although dressing (unquenching) the quark model is complicated, it is in principle a solvable problem. This will require careful definition and checking of the procedures for extracting poles from energy-dependent partial waves or directly from partial-wave data.

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