Physics of Atomic Nuclei, Vol. 65, No. 3, 2002, pp. 539–545. Translated from Yadernaya Fizika, Vol. 65, No. 3, 2002, pp. 566–572.




# Determination of the Parameters of the $\Delta(1232)$ Resonance from Partial-Wave Analyses of Elastic $\pi N$ Scattering

**A. S. Omelaenko***

*Kharkov Institute for Physics and Technology, Akademicheskaya ul. 1, Kharkov, 310108 Ukraine*
Received November 1, 2000; in final form, February 15, 2001

**Abstract**—On the basis of data on the $P_{33}$ amplitude from various partial-wave analyses of elastic $\pi N$ scattering, the pole characteristics of the $\Delta(1232)$ resonance are determined within the resonance model. An approximate analytic formula that relates the residue to the background is obtained. Estimates confirm that the nonresonance part of the phase shift is small and differs significantly from the results of the calculations within the current algebra and the approach of effective Lagrangians. This contradiction is removed in a modified resonance-model version developed on the basis of taking into account the quadratic term in the expansion of the Jost function in a series at the pole point. It is shown that the coordinates of the pole and the phase shift of the residue change only slightly in relation to the results within the traditional model, but that the absolute value of the residue increases by about 20%.


## 1. INTRODUCTION

Although elastic $\pi N$ scattering has been studied for a long time and is thought to be well understood, there is a significant scatter of the results for the absolute value and the phase shift of the residue at the pole of the $P_{33}$-wave amplitude in the $\Delta(1232)$-resonance region (Table 1, [1, 2]). In principle, this can be due to modifications in the experimental basis, its qualitative and quantitative improvements. Another reason behind this can be associated with the presence of a model ambiguity in the analytic description of the amplitude. This is manifested significantly in the extrapolation of the amplitude toward the pole point occurring beyond the physical region. As to the $P_{33}$ amplitude itself, it is well described within the resonance model [3, 4], where an additional background contribution is taken into account along with the standard Breit–Wigner expression. From the results of the calculations presented in [4], it follows that the magnitude of this background is moderate and that the corresponding phase shift at the resonance point is about 3° to 4°. However, this estimate contradicts the results of the calculations within the current algebra and the approach of effective Lagrangians, where the background phase shift at the resonance proves to be about 15° [5, 6].

In order to clarify these questions, the resonance and the pole characteristics of the $P_{33}$-wave amplitude are calculated here within a realistic resonance model by using data from a few partial-wave analyses of elastic $\pi N$ scattering. The relation between the residues of the total and the resonance amplitude is derived in an analytic form with allowance for the background, and the resonance model is modified by taking into account the second-order correction in the expansion of the Jost function in the vicinity of the pole of the amplitude.

*e-mail: omelaenko@kipt.kharkov.ua

## 2. AMPLITUDE OF THE $P_{33}$ WAVE IN THE $\Delta(1232)$-RESONANCE REGION

### 2.1. Residue in the Resonance Model

In the region of the excitation of the first nucleon resonance, the description of the $P_{33}$-wave amplitude for $\pi N$ scattering is simplified because this amplitude is elastic. In the resonance model, the relevant $S$-matrix element depending on the total energy $W$ (in the c.m. frame) has the factorized form

$$S(W) = S_B(W)S_R(W), \quad (1)$$

where the quantities $S_B$ and $S_R$ correspond to the background and the resonance, respectively; in the physical region, they are determined by corresponding real phase shifts,

$$S = e^{2i\delta}, \quad S_B = e^{2i\delta_B}, \quad S_R = e^{2i\delta_R}. \quad (2)$$

Here, the rule of phase summation is

$$\delta(W) = \delta_B(W) + \delta_R(W). \quad (3)$$

In the complex plane of $W$, the quantities $S_B$ and $S_R$ are given by

$$S_B(W) = \frac{1 + iB(W)}{1 - iB(W)}, \quad (4a)$$

**Table 1.** Pole parameters of the $P_{33}$-wave amplitude [1]

| Data | $M_P$, MeV | $\Gamma_P/2$, MeV | $\lvert\text{res}\rvert$, MeV | $\varphi(\text{res})$, deg |
|---|---|---|---|---|
| Cutkosky 80 | $1210 \pm 1$ | $53 \pm 2$ | $50 \pm 1$ | $-47 \pm 1$ |
| Arndt 91 | 1210 | 50 | 52 | −31 |
| Hoehler 93 | 1209 | 50 | 50 | −48 |
| Arndt 95 | 1211 | 50 | 38 | −22 |
| Arndt 99a | 1211 | 51 | 39 | −23 |
| Arndt 99b | 1211 | 50 | 47 | −47 |

Note: The symbols $\lvert\text{res}\rvert$ and $\varphi(\text{res})$ stand for, respectively, the absolute value and the phase shift of the residue at the pole point $M_P - i\Gamma_P/2$ in the complex plane of the total energy $W$. The values in the two lower rows correspond to versions of the treatment of the data from the analysis reported in [2] (private communication of Arndt).

$$S_R(W) = \frac{W - M_R - i\Gamma(W)/2}{W - M_R + i\Gamma(W)/2}. \tag{4b}$$

It is assumed that the background function $B(W)$ introduced above and the width $\Gamma(W)$ satisfy the conditions $B(W) = B^*(W^*)$ and $\Gamma(W) = \Gamma^*(W^*)$, so that, in the physical region, $B(W)$ and $\Gamma(W)$ are real and are related to the phase shifts $\delta_B$ and $\delta_R$ as

$$\tan\delta_B(W) = B(W), \tag{5a}$$

$$\tan\delta_R(W) = \frac{\Gamma(W)/2}{M_R - W}, \tag{5b}$$

where $M_R$ is the resonance mass. From Eq. (4b), it can be seen that, at the $W$ values that satisfy the equation

$$W - M_R + i\Gamma(W) = 0, \tag{6}$$

the function $S_R$ has a pole. According to scattering theory, the root of Eq. (6), $W_P = M_P - i\Gamma_P/2$, that lies in the fourth quadrant of the complex plane of $W$ at positive values of $M_P$ and $\Gamma_P$ corresponds to a resonance. Considering that the expansion of the denominator on the right-hand side of Eq. (4b) in a series in powers of $(W - W_P)$ begins from the linear term and retaining only this term in the vicinity of the pole, we can recast (4b) into the form

$$S_R(W) \cong \frac{W - M_R - i\Gamma(W)/2}{(1 + i\Gamma'(W_P)/2)(W - W_P)}, \tag{7}$$

where $\Gamma'(W_P)$ is the value of the derivative $d\Gamma(W)/dW$ at the pole point. From Eqs. (6) and (7), we can obtain an exact formula for the residue:

$$\text{res}(S_R) = \frac{-i\Gamma(W_P)}{1 + i\Gamma'(W_P)/2}. \tag{8}$$

We will now consider that the numerator on the right-hand side of (7) vanishes at the point $W_P^* = M_P + i\Gamma_P/2$, which is conjugate to the pole, and expand it in a series at this point. Retaining only the first term in this expansion and assuming that this approximation can be used at the pole as well, we find that, in the unipolar approximation, the resonance $S$-matrix element has the form

$$S_R(W) \cong \frac{1 - i\Gamma'(W_P^*)/2}{1 + i\Gamma'(W_P)/2}\,\frac{W - M_P - i\Gamma_P/2}{W - M_P + i\Gamma_P/2}, \tag{9}$$

whence we obtain an approximate expression for the residue:

$$\text{res}(S_R) \cong -i\Gamma_P\frac{1 - i\Gamma'(W_P^*)/2}{1 + i\Gamma'(W_P)/2}. \tag{10}$$

For expression (10) to be correct, it is obviously necessary that the distance $\Gamma_P$ between $W_P^*$ and the pole not be overly large and that the energy dependence of the width $\Gamma(W)$ be sufficiently smooth. Comparing (8) and (10), we also obtain

$$\Gamma(W_P)/\Gamma_P \cong 1 - i\Gamma'(W_P^*)/2. \tag{11}$$

In particular, the equality $\Gamma(W_P) = \Gamma_P$ holds for an energy-independent width.

The residue of the amplitude $T \equiv (S-1)/2i$ is usually presented in analyses. It follows from (10) that the absolute value and the phase shift of the residue of the resonance amplitude are

$$|\text{res}(T_R)| \cong \Gamma_P/2, \tag{12}$$

$$\varphi(\text{res}(T_R)) = 2\varphi_0, \tag{13}$$

where

$$\varphi_0 \cong \arg(1 - i\Gamma'(W_P^*)/2) \tag{14}$$
$$= -\arg(1 + i\Gamma'(W_P)/2).$$

From (13) and (14), it can be seen that the phase shift of the residue is equal to zero if the width is independent of energy.

In general, the approximate result for the residue of the amplitude with allowance for the background has the form (it should be noted that the presence of a background does not affect the coordinates of the pole)

$$|\mathrm{res}(T)| \cong (\Gamma_P/2)|S_B(W_P)|, \tag{15}$$

$$\varphi(\mathrm{res}(T)) \cong 2\varphi_0 + \arg(S_B). \tag{16}$$

The second term on the right-hand side of (16) determines the contribution of the background to the phase shift of the residue.

### *2.2. Inclusion of the Second-Order Correction in the Expansion of the Jost Function*

As a matter of fact, the above approximation is a starting point for constructing, in a standard way, the resonance model, where only the first term in the expansion of the Jost function in a power series in the vicinity of the pole point is taken into account (see, for example, [7]). By introducing the parameter $c$ defined by the ratio of the coefficients of the second and the first term in this expansion, we can take into account the second-order correction on the right-hand side of (9) by making the substitution

$$\frac{W - M_P - i\Gamma_P/2}{W - M_P + i\Gamma_P/2}$$

$$\to \frac{W - M_P - i\Gamma_P/2 + c^*(W - M_P - i\Gamma_P/2)^2}{W - M_P + i\Gamma_P/2 + c(W - M_P + i\Gamma_P/2)^2}.$$

Upon going over to the model and introducing the energy-dependent width in (4b), there arises the additional factor

$$S_c(W) = \frac{1 + c^*(W - M_R - i\Gamma(W)/2)}{1 + c(W - M_R + i\Gamma(W)/2)}. \tag{17}$$

For the corresponding phase shift $\delta_c$ determined by the relation $S_c(W) = e^{2i\delta_c}$ to vanish as we approach the threshold, the constant $c$ must be real. In this case, the modification of the model via the inclusion of the second-order term in the expansion of the Jost function generates, in the phase shift, the additional contribution given by

$$\tan\delta_c = -\frac{c\Gamma(W)/2}{1 + c(W - M_R)}. \tag{18}$$

Thus, the total phase shift is equal to the sum

$$\delta = \delta_B + \delta_R + \delta_c, \tag{19}$$

while the expressions for the absolute value of the residue and for its phase shift [(15) and (16), respectively] are modified to become

$$|\mathrm{res}(T)| \cong (\Gamma_P/2)|S_B(W_P)||1 - ic\Gamma(W_P)|, \tag{20}$$

$$\varphi(\mathrm{res}(T)) \cong 2\varphi_0 \tag{21}$$
$$+ \arg(S_B(W_P)) + \arg(1 - ic\Gamma(W_P)).$$

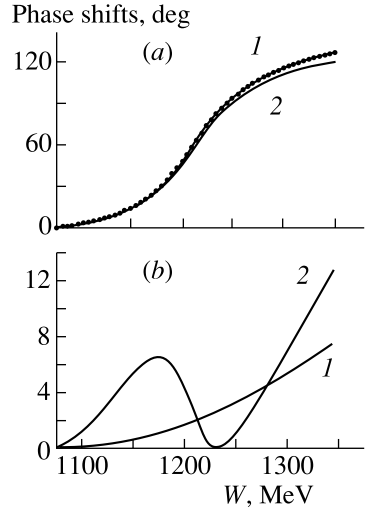


**Fig. 1.** Energy dependence of the total phase shift $\delta$ and of its resonance and background parts according to the calculations within the standard resonance model at $M_R = 1235.14$ MeV, $\Gamma_R = 123.36$ MeV, $r = 0.97520$ fm, and $a = 0.02822$ fm$^3$: ($a$) results of the calculations for $\delta$ and $\delta_R$ (curves *1* and *2*, respectively) and data of the SM99 partial-wave analysis (points); ($b$) results of the calculations for the background phase shift $\delta_B$ and for the difference of $\delta_R$ and the values of this quantity at $r = 0$ (curves *1* and *2*, respectively).

## 3. NUMERICAL CALCULATIONS

### *3.1. Resonance Model*

The width was calculated here by the formula

$$\Gamma(W) = \Gamma_R(q/q_R)^3 R(W), \tag{22}$$

which takes into account the threshold dependence and which involves the c.m. particle momentum $q \equiv q(W)$; its value at $W = M_R$, $q_R$; the quantity $\Gamma_R = \Gamma(M_R)$; and the factor $R(W)$ correcting the energy dependence of the width,

$$R(W) = (1 + q_R^2 r^2)/(1 + q^2 r^2), \tag{23}$$

with $r$ being a phenomenological parameter. The background was described with the aid of Eqs. (4a) and (5a), where $B(W)$ was parametrized as

$$B(W) = aq^3(W). \tag{24}$$

The inclusion of the additional factor $2M_R/(M_R + W)$ in expression (22) for the width leads to the relativistic version of the model, with the resonance amplitude being given by

$$T_R(W) = \frac{\Gamma(W)W_R}{M_R^2 - W^2 - i\Gamma(W)M_R}. \tag{25}$$

However, the calculations have revealed that, in this case, the results for the resonance and pole parameters remain virtually unchanged. For this reason, we will employ below only the nonrelativistic formulation of the model.

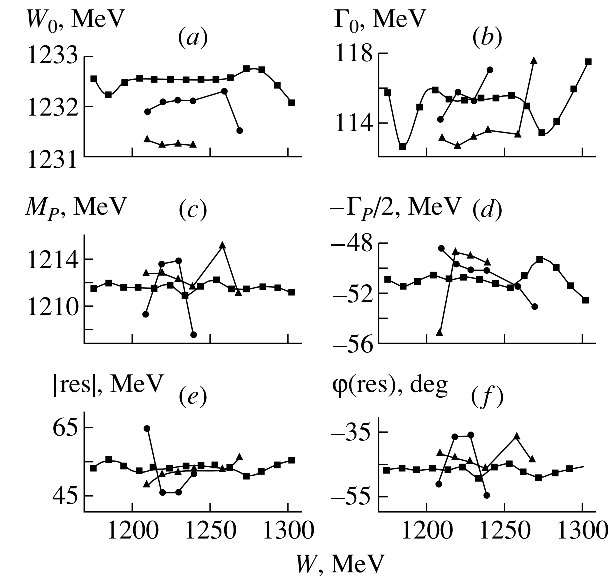


**Fig. 2.** Resonance and pole parameters according to the data of the (boxes) SM99, (triangles) SM99s, and (circles) KA84 partial-wave analyses in the energy intervals of widths 30, 60, and 80 MeV, respectively, their centers being plotted along the abscissa.

### *3.2. Fitting Data on the Phase Shift of the $P_{33}$ Amplitude*

The model parameters $M_R$, $\Gamma_R$, $r$, and $a$ were determined from the data on the phase shift of the $P_{33}$ wave in the region of $W$ from the threshold to 1350 MeV that were obtained with a step of 5 MeV from the SAID system available through the Internet (http://said.phys.vt.edu). It includes the parametrized (SM99) and the energy-independent (SM99s) version of the partial-wave analysis reported in [2], its preceding versions SM90 [8] and SM95 [9], and the KP80 partial-wave analysis [10] and its smoothed version KA84. Upon fitting the parameters, the resonance model makes it possible to obtain a good description of the data on $\delta$ in all cases. By way of example, the result of a fit to data of the SM99 partial-wave analysis is presented in Fig. 1*a*. Figure 1*b* illustrates the role that the empirical parameter $r$ plays in the formation of the resonance contribution. The background phase shift is positive, taking values of 2° to 3° at the resonance point.

The calculations were further performed for the resonance parameters $M_0$ (the position of the point where the phase shift takes the value of 90°) and $\Gamma_0 = 2/(d\delta/dW)|_{W=M_0}$ (the experimental width) and for the pole parameters—namely, the coordinates of the pole of the amplitude, the absolute value of the corresponding residue, and its phase shift $\varphi[\text{res} \equiv \text{res}(T)]$. The resonance and pole parameters for the SM99 partial-wave analysis and for solutions of other partial-wave analyses are quoted in Table 2. There, the values of $\chi^2$ are of a rather arbitrary character because, as a rule, the errors in the phase-shift values are not presented in partial-wave analyses—in the calculations, the experimental points were arbitrarily assigned the error values of 0.25°. The SM99s (single energy) version, which was implemented for 16 energy values in such a way that a strong dependence on the experimental data that fell within the vicinity of each node was preserved, is the only partial-wave analysis presenting the error values. The value of $\chi^2 = 65$ was obtained from a fit to this solution, with the overwhelming contribution to it coming from the point at $W = 1180$ MeV. Upon the elimination of this point, $\chi^2$ decreased to 27, but the results for the sought parameters changed only slightly. Unfortunately, there are only a few points in the central region, which is of greatest interest.

In order to estimate the effect exerted by individual groups of experimental points on the formation of the values of the sought parameters, fits were also performed for data from specific energy intervals. In principle, the results obtained in this way must be compatible if the experimental data are of a very high quality and if the model used in fitting is adequate. For the SM99, SM99s, and KA84 partial-wave analyses, this was so in some cases for energy intervals of widths 30, 60, and 80 MeV, respectively (Fig. 2). It can be seen that the region around 1210–1220 MeV is the most informative. For the SM99 solution, this test yielded the best result—at the center of the resonance distribution, the results for the experimental mass, the width, and all of the pole parameters showed the weakest dependence on the choice of input data interval, errors increasing away from the center of the distribution. For the energy-independent SM99s solution and the earlier KA84 solution, there are pronounced deviations in the central region that is shown in the graphs, the results losing physical significance beyond it. In view of this, the resonance and pole parameters were determined here on the basis of only data in the central region ($1180 < W < 1260$ MeV), since this region is the most appropriate for this purpose. In Table 2, the corresponding results are presented in the lower row for each of the analyses considered in the present study. It can be seen that, in some cases, these results differ from those obtained from a fit over the entire resonance region, especially for the phase shift of the residue.

### *3.3. Approximate Formula for the Residue*

The approximate values calculated by formulas (14)–(16) for the absolute value and the phase shift of the residue within the standard resonance model are presented parenthetically in Table 2 under the precise

**Table 2.** Resonance and pole parameters of $P_{33}$-wave amplitude

| Data | $N$ | $\chi^2$ | $M_0$, MeV | $\Gamma_0$, MeV | $M_P$, MeV | $\Gamma_P/2$, MeV | \|res\|, MeV | $\varphi$(res), deg |
|---|---|---|---|---|---|---|---|---|
| KP80 | 27 | 19.0 | 1231.0 | 116.0 | 1209.2 | 50.4 | 52.4 | −48.9 |
| | | | | | | | (52.9) | (−49.9) |
| * | 10 | 4.2 | 1230.9 | 115.4 | 1210.1 | 49.5 | 50.1 | −46.2 |
| KA84 | 55 | 54.0 | 1231.2 | 118.2 | 1208.7 | 51.4 | 53.7 | −49.8 |
| | | | | | | | (54.5) | (−51.1) |
| * | 17 | 7.9 | 1231.2 | 116.8 | 1211.2 | 50.7 | 51.3 | −43.2 |
| SM90 | 55 | 0.25 | 1231.3 | 113.9 | 1210.6 | 49.9 | 51.8 | −46.8 |
| | | | | | | | (51.8) | (−46.9) |
| * | 17 | 0.14 | 1231.3 | 114.0 | 1210.5 | 49.8 | 51.6 | −46.9 |
| SM95 | 55 | 0.18 | 1231.9 | 113.0 | 1211.6 | 50.1 | 52.4 | −46.0 |
| | | | | | | | (52.4) | (−46.2) |
| * | 17 | 0.08 | 1232.0 | 113.2 | 1211.6 | 50.2 | 52.5 | −46.2 |
| SM99 | 55 | 0.15 | 1232.5 | 115.2 | 1211.5 | 50.8 | 53.1 | −46.9 |
| | | | | | | | (53.2) | (−47.1) |
| * | 17 | 0.00 | 1232.5 | 115.4 | 1211.5 | 50.9 | 53.2 | −47.0 |
| SM99s | 16 | 65.0 | 1232.1 | 114.7 | 1210.9 | 50.5 | 52.9 | −48.0 |
| | | | | | | | (53.6) | (−49.2) |
| SM99s, c | 15 | 27.0 | 1232.1 | 115.0 | 1210.6 | 50.5 | 53.0 | −48.6 |
| * | 5 | 0.82 | 1231.9 | 112.8 | 1212.9 | 48.3 | 47.6 | −42.0 |

Note: $N$ is the number of points; asterisks and the letter "c" correspond to data for $1180 < W < 1260$ MeV and data without the point at $W = 1180$ MeV, respectively; and the values in parentheses correspond to the calculations of the residue by formulas (14)–(16).

**Table 3.** Resonance and pole parameters of the $P_{33}$-wave amplitude within the modified resonance model

| Data | $M_0$, MeV | $\Gamma_0$, MeV | $M_P$, MeV | $\Gamma_P/2$, MeV | \|res\|, MeV | $\varphi$(res), deg |
|---|---|---|---|---|---|---|
| KP80 | 1231.1 | 118.8 | 1206.7 | 52.2 | 57.0 | −55.4 |
| KA84 | 1231.4 | 120.8 | 1206.4 | 53.2 | 58.5 | −56.1 |
| SM99 | 1232.8 | 118.2 | 1209.1 | 53.6 | 60.7 | −53.5 |
| SM99s | 1233.3 | 117.9 | 1210.4 | 55.2 | 64.7 | −51.7 |

results and are in good agreement with them. Also, there is the possibility of obtaining, within a realistic model, an independent estimate of the background in the case where data on the pole parameters of the resonance are available. Indeed, formula (15) makes it possible to establish a relation between the relative value of the residue, $y$, and the background parameter $a$:

$$y \equiv \frac{|\mathrm{res}(T)|}{\Gamma_P/2} \cong \left|\frac{1 + iaq^3(W_P)}{1 - iaq^3(W_P)}\right|. \qquad (26)$$

Upon solving Eq. (26) for $a$, the background phase shift $\delta_B(W)$ can be calculated with the aid of (24) and (5a). In order to characterize the background, the results of such a calculation at $W = 1232$ MeV are presented in Fig. 3 versus $y$ from various partial-wave analyses. It can be seen that the estimates of the background phase shift at the resonance are grouped around the values of $-15°$, $-3°$, and $+3°$. In order to obtain the value of $+15°$, which is characteristic of the approach of effective Lagrangians [4], one must have the value of $y \approx 1.32$, which would correspond to the absolute value of the residue of about 68 MeV. The contribution of the background to the phase shift

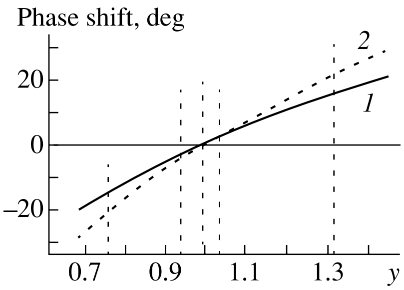


**Fig. 3.** Background phase shift $\delta_B$ at $W = 1232$ MeV (curve *1*) and contribution of the background to the phase shift of the residue (curve *2*) versus $y$. Vertical dashed lines correspond to ($y \approx 0.76$) Arndt 95, Arndt 99a data; ($y \approx 0.94$) Cutkosky 80, Arndt 99b data; ($y \approx 1.00$) Hoehler 93 data; ($y \approx 1.04$) SM90 data and the present estimate based on the SM99 data; and ($y \approx 1.32$) $\delta_B(1232) \approx 15°$.

of the residue also changes considerably with $y$ (see curve *2* in Fig. 3).

### *3.4. Modified Resonance Model*

Alternatively, the phase shift $\delta$ was described with the aid of formulas (17)–(21), which were derived within the model developed by taking into account the second-order term in the expansion of the Jost function and by introducing a free parameter $c$. As before, the background was described with the aid of (24); however, the empirical factor $R(W)$ (with the parameter $r$), which is typical of the standard model, was not introduced in expression (22) for the width. Hence, the modified model involves four adjustable parameters as before. A good description was obtained for data on $\delta$ from all of the partial-wave analyses considered here. By way of example, the result for the SM99 partial-wave analysis is presented in Fig. 4*a*. At the resonance point, we have $\delta_B \sim 23°$ in this case. However, the correction associated with the inclusion of the second-order term in the expansion of the Jost function is negative, and the total nonresonance phase shift approaches 15° (Fig. 4*b*). This is in agreement with the estimates obtained within the approach of effective Lagrangians [6]. By and large, the curve representing the nonresonance phase shift is in reasonable agreement with the estimate based on the current-algebra model [5] as well. The values of the resonance and pole parameters for the modified model are presented in Table 3. In relation to what we have in the standard resonance model, the coordinates of the pole exhibit moderate shifts that depend on the choice of analysis, while the absolute value of the residue appears to be about 20% greater. The phase shift of the residue increases somewhat. As to the experimental mass and width, the results for them are close, on the whole, to traditional values.

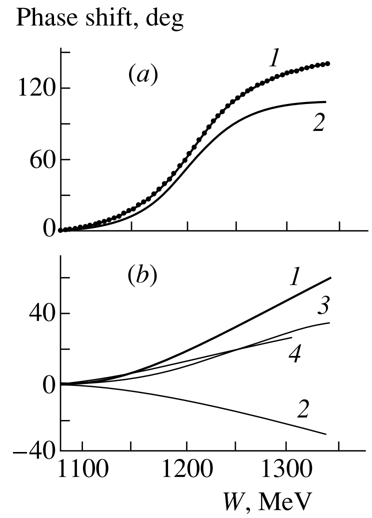


**Fig. 4.** Energy dependence of the total phase shift $\delta$ and of its resonance and background parts within the modified resonance model at $M_R = 1256.08$ MeV, $\Gamma_R = 222.44$ MeV, $a = 0.29143$ fm$^3$, and $c = 0.34821$ fm: (*a*) results for $\delta$ and $\delta_R$ (curves *1* and *2*, respectively) and data of the SM99 partial-wave analysis (points); (*b*) results for $\delta_B$, $\delta_c$, and $\delta_B + \delta_c$ (curves *1*, *2*, and *3*, respectively) and nonresonance phase shift borrowed from [3] (curve *4*).

## 4. CONCLUSIONS

The basic points of the present study and its conclusions concerning the $P_{33}$-wave amplitude for $\pi N$ scattering can be briefly formulated as follows:

(i) As a matter of fact, the approximation considered in Section 2 is the starting point in constructing the resonance model on the basis of the expansion of the Jost function in the vicinity of the pole of the amplitude; the above test has demonstrated its applicability to calculating the pole parameters. For the absolute value and the phase shift of the residue, the analytic expressions (14)–(16) have been derived in this approximation. From these expressions, it follows, among other things, that the equality of the absolute values of the residue and the imaginary pole coordinate suggests the absence of background and that the resonance contribution to the phase shift of the residue is determined by the value of $d\Gamma(W)/dW$ at the pole point.

(ii) If the background is described by one parameter and if the residue and the pole coordinates are known, the background can be estimated on the basis of expression (15) without invoking information about $\Gamma(W)$ for this. Estimates of the background phase shift according to data from various partial-wave analyses exhibit a wide scatter—from $-15°$ to $+3°$ at the resonance point; however, values of about $+15°$, which are characteristic of the calculations within the current algebra and the approach of effective Lagrangians [5, 6], cannot be obtained in this way.

(iii) A retrospective fit to the data from basic partial-wave analyses has revealed that the pole parameters have changed only slightly over the last two decades. This may indirectly suggest that advances in experiments studying the Δ(1232)-resonance region are not as pronounced as might have been expected.

(iv) The present calculations within the constrained energy intervals has enabled us to assess the "energy resolution" of various analyses. For example, a determination of the resonance and pole parameters on the basis of the SM99 analysis, which employs the most comprehensive sample of experimental data, is possible by using the data from the interval of width about 30 MeV. For other analyses, the minimal width of such an interval varies from 60 to 80 MeV, and the results for the pole parameters change significantly in response to its shift. This demonstrates that the region of the first resonance has not yet received adequately study, so that new systematic and precise measurements are required for performing a reliable analysis.

(v) A modified resonance-model version developed on the basis of retaining the quadratic term in the expansion of the Jost function at the pole point has been considered. A satisfactory description of data from partial-wave analyses has been obtained. The resulting estimate of the nonresonance phase shift complies well with the results of the calculations within the current algebra and the approach of effective Lagrangians. In relation to the results within the traditional model, the absolute value of the residue undergoes the most pronounced change (an increase of about 20%).

## ACKNOWLEDGMENTS

I am grateful to I.I. Strakovsky for a discussion and enlightening comments and to R.A. Arndt for kindly placing the data on the residue at my disposal prior their publication. I also acknowledge the crucial importance of the SAID information system.

## REFERENCES

1. Particle Data Group, Eur. Phys. J. C **15**, 725 (2000).
2. R. A. Arndt, R. L. Workman, I. I. Strakovsky, and M. M. Pavan, nucl-th/9807087; submitted to Phys. Rev. C.
3. F. Nichitiu, *Analiza de Fasa in Fizica Interactiilor Nucleare* (Editura Akademiei RSR, Bucharest, 1980; Mir, Moscow, 1983).
4. S. S. Vasan, Nucl. Phys. B **106**, 71 (1976).
5. M. G. Olsson, Phys. Rev. D **13**, 2502 (1976).
6. R. M. Davidson, N. S. Mukhopadhyay, and R. S. Wittman, Phys. Rev. D **43**, 71 (1991).
7. A. G. Sitenko, *Scattering Theory* (Vishcha Shkola, Kiev, 1975; Springer-Verlag, Berlin, 1991).
8. R. A. Arndt, L. Zhujun, L. D. Roper, *et al.*, Phys. Rev. D **43**, 2131 (1991).
9. R. A. Arndt, I. I. Strakovsky, R. L. Workman, and M. M. Pavan, Phys. Rev. C **52**, 2120 (1995).
10. R. Koch and E. Pietarinen, Nucl. Phys. A **336**, 331 (1980).

*Translated by A. Isaakyan*