CLASSICAL ELECTRODYNAMICS II

Homework Set 2

January 31, 2020

1. In class, we considered how Maxwell's equations in a medium would be modified if magnetic monopoles existed:

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho_e, \\ \nabla \cdot \mathbf{B} &= \rho_m, \\ \nabla \times \mathbf{H} &= \mathbf{J}_e + \frac{\partial \mathbf{D}}{\partial t}, \\ \nabla \times \mathbf{E} &= -\mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t}. \end{aligned}$$

   Show explicitly that these equations are invariant under a duality transformation, as defined in class.

2. Consider electromagnetic plane waves propagating in a medium in which $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$, where the fields are given by the complex representation:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)} \, , \\ \mathbf{B} &= \mathbf{B}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)} \, , \end{aligned}$$

   with $\mathbf{E}_0$ and $\mathbf{B}_0$ complex. Show explicitly that the time-averaged Poynting vector is given by

$$\mathbf{S} = \frac{1}{2} \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \, .$$