

CLASSICAL ELECTRODYNAMICS II

Homework Set 3

February 7, 2020

1. Consider a monochromatic plane wave propagating along the z axis in an isotropic nonpermeable ($\mu = \mu_0$) dielectric. If the dielectric is a gyrotropic material that has been placed in a static external magnetic field, then the electric displacement vector can be written as

$$\mathbf{D} = \epsilon \mathbf{E} + i \mathbf{E} \times \mathbf{g} ,$$

where the permittivity ϵ is a positive real number and \mathbf{g} is a constant real vector (called the gyration vector), which is proportional to the applied magnetic field. If the applied magnetic field is along the direction of propagation, then $\mathbf{g} = g \hat{z}$. The index of refraction for the medium can be written as $n = ck/\omega$, where ω is the frequency of the propagating wave and k is its wave number. Show that this material is birefringent (double refracting) with two indices of refraction and determine their values.

2. Consider electromagnetic waves in source-free space where $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Given the explicitly real field \mathbf{E} for each part below, calculate the corresponding magnetic induction \mathbf{B} , the Poynting vector, $\mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0$, and the time-averaged Poynting vector. Interpret each case using, as appropriate, the following descriptors: traveling wave; standing wave; plane wave; spherical wave; linearly polarized wave; circularly polarized wave; elliptically polarized wave. The time-averaged Poynting vector is zero for a standing wave.

(a) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t)$

(b) $\mathbf{E} = \mathbf{E}_0 \sin(kr - \omega t)$

(c) $\mathbf{E} = \mathbf{E}_0 \sin(\mathbf{k} \cdot \mathbf{r}) \sin(\omega t)$

(d) $\mathbf{E} = E_0 \hat{x} \cos(kz - \omega t) + E_0 \hat{y} \sin(kz - \omega t)$

(e) $\mathbf{E} = 3E_0 \hat{x} \cos(kz - \omega t) + 2E_0 \hat{y} \sin(kz - \omega t)$

(f) $\mathbf{E} = E_0 [\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t)] \sin(kz)$