CLASSICAL ELECTRODYNAMICS II

Homework Set 4

February 17, 2020

1. Consider a material in which the half-space with $z > 0$ is filled with a medium with permeability μ' and dielectric constant ϵ', and the half-space with $z < 0$ is filled with a medium with permeability μ and dielectric constant ϵ. A electromagnetic wave with electric field,

$$\mathbf{E} = \mathbf{E}_0 \, \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega \mathrm{t})} \ ,$$

is incident on the interface at $z = 0$ between the two dielectrics. Consider the case where $\mathbf{E}$ is parallel to the plane of incidence. Then, starting with the boundary conditions on the fields as derived in class, derive expressions for the ratios E_0'/E_0 and E_0''/E_0, where E_0' is the amplitude of the electric field of the transmitted wave and E_0'' is the amplitude of electric field of the reflected wave.

2. The reflection and transmission coefficients, R and T, respectively, are defined as

$$R = \left| \frac{(\mathbf{S}'' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| \qquad T = \left| \frac{(\mathbf{S}' \cdot \hat{z})}{(\mathbf{S} \cdot \hat{z})} \right| \ ,$$

where $\mathbf{S}$, $\mathbf{S}'$, and $\mathbf{S}''$ are the time-averaged Poynting vectors for the incident, transmitted, and reflected waves, respectively.

   (a) Calculate R and T in terms of the general ratios E_0'/E_0 and E_0''/E_0.

   (b) Next calculate R and T for the particular case described in problem 1.

   (c) Finally use your result from part (b) to show that $R + T = 1$.