CLASSICAL ELECTRODYNAMICS II

Homework Set 5

February 24, 2020

1. A circularly polarized plane wave moving in the z direction has a finite extent in the x and y directions. Assuming that the amplitude modulation is slowly varying (the wave is many wavelengths broad), show that the electric and magnetic fields are given approximately by

$$\begin{aligned} \mathbf{E}(x,y,z,t) &\approx \left[E_0(x,y)(\hat{x} \pm i\hat{y}) + \frac{i}{k}\left(\frac{\partial E_0}{\partial x} \pm i\frac{\partial E_0}{\partial y}\right)\hat{z}\right] \mathrm{e}^{ikz-i\omega t} \,, \\ \mathbf{B} &\approx \mp i\sqrt{\mu\epsilon}\;\mathbf{E} \,. \end{aligned}$$

1. In class, I derived the dispersion relation:

$$\mathrm{Re}\ \epsilon(\omega) = 1 + \frac{2}{\pi}\ P\int_0^\infty \frac{\omega'\ \mathrm{Im}\ \epsilon(\omega')}{\omega'^2 - \omega^2}\, d\omega' \,.$$

   Use this dispersion relation to calculate Re $\epsilon(\omega)$, given the imaginary part of $\epsilon(\omega)$ for positive ω as

   (a)

$$\mathrm{Im}\ \epsilon(\omega) = \lambda\ [\theta(\omega-\omega_1) - \theta(\omega-\omega_2)] \,,$$

   where $\omega_2 > \omega_1 > 0$. Here $\theta(\tau)$ is the step function defined such that $\theta(\tau) = 0$ for $\tau < 0$ and $\theta(\tau) = 1$ for $\tau > 0$.

   (b) Repeat the calculation using the single-resonance form,

$$\mathrm{Im}\ \epsilon(\omega) = \frac{\lambda\gamma\omega}{(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2} \,.$$