NUCLEAR PHYSICS

Homework Set 3

September 29, 2006

1. In elastic scattering, the outgoing flux of particles must equal the incoming flux, with the result that the phase shift $\delta(k)$ is real. This implies that the scattering "matrix" $S(k) = \exp(2i\delta(k))$ is unitary; that is,

$$S^*(k) = \frac{1}{S(k)} \qquad \text{for real } k \ .$$

One of the simplest forms of the S-matrix obeying unitarity is a rational function of k,

$$S(k) = \left(\frac{k+i\beta}{k-i\beta}\right) \cdot \left(\frac{k-i\alpha}{k+i\alpha}\right) \ ,$$

where we assume α and β to be real and positive. For sufficiently small k, we may represent $k \cot \delta(k)$ by the *effective range approximation*:

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 k^2 \ .$$

(This is just an expansion of $k \cot \delta(k)$ in powers of k^2 in which higher powers of k^2 have been neglected.)

(a) Find a and r_0 for the case in which $S(k)$ has the form above. In particular, show that the effective range approximation gives an *exact* description of δ for all k with

$$a = -\left(\frac{\beta-\alpha}{\alpha\beta}\right) \ , \qquad r_0 = \frac{2}{\beta-\alpha} \ .$$

(b) For S-wave n-p scattering, the singlet scattering length and effective range have the values $a = -23.7$ fm and $r_0 = 2.7$ fm. Determine the corresponding values of α and β.

A potential that exactly reproduces this scattering matrix for all k is the Bargmann potential [V. Bargmann, Rev. Mod. Phys. **21**, 488 (1949)], given by

$$V(r) = 2\beta^2(\alpha^2-\beta^2)(\beta \cosh \beta r + \alpha \sinh \beta r)^{-2} \ ,$$

which asymptotically behaves as $\exp(-2\beta r)$.

2. For a velocity-independent N-N potential, the only two-body scalars that can be formed of $\mathbf{r}$, $\mathbf{S} = (\vec{\sigma_1} + \vec{\sigma_2})/2$, and $\mathbf{T} = (\vec{\tau_1} + \vec{\tau_2})/2$ are r, $\vec{\sigma_1} \cdot \vec{\sigma_2}$, $\vec{\tau_1} \cdot \vec{\tau_2}$, $(\vec{\sigma_1} \cdot \vec{\sigma_2})(\vec{\tau_1} \cdot \vec{\tau_2})$, and $S_{12} = 3(\vec{\sigma_1} \cdot \mathbf{r})(\vec{\sigma_2} \cdot \mathrm{r})/r^2 - \vec{\sigma_1} \cdot \vec{\sigma_2}$.

   (a) Show that $(\mathbf{r} \times \mathbf{S}) \cdot (\mathbf{r} \times \mathbf{S})$ can be reduced to functions of the scalars above.

   (b) The helicity operator is defined as $\mathcal{H} = (\mathbf{S} \cdot \mathbf{r})/r$. Give the symmetry argument(s) why the N-N potential does not contain a term proportional to the helicity operator.

3. Neglecting the pairing term, the mass of a nucleus with Z protons and A nucleons is given approximately by the semi-empirical mass formula,

$$M(Z, A) = ZM_p + (A - Z)M_n - a_v A + a_s A^{2/3} + \frac{a_c Z(Z-1)}{A^{1/3}} + \frac{a_a (A - 2Z)^2}{A} .$$

   The proton separation energy S_p is defined as the energy needed to remove a proton from a nucleus:

$$S_p = M_p + M(Z-1, A-1) - M(Z, A) .$$

   Show that for large Z and A, the proton separation energy is approximately given by

$$\begin{aligned} S_p &= a_v - \frac{2a_s}{3A^{1/3}} + \frac{a_c Z(Z-1)}{3A^{4/3}} - a_a \left[1 - \left(\frac{2Z}{A}\right)^2\right] \\ &- \frac{a_c(2Z-1)}{A^{1/3}} + 4a_a \left(1 - \frac{2Z}{A}\right) . \end{aligned}$$

   (*Hint:* Use a differential approach.)