NUCLEAR PHYSICS

Homework Set 4

October 13, 2006

1. The expectation value of the total kinetic energy of the proton and neutron in the deuteron is given by

$$\langle T\rangle = -\frac{\hbar^2}{2m}\int_0^\infty \psi^* \nabla^2 \psi \; 4\pi r^2 \, dr \; ,$$

where m is the reduced mass of the proton and neutron system and $\psi(r)$ is the deuteron wave function. If only $L = 0$ contributes, we may write $\psi(r) = [u(r)/r]\; Y_{00}$, with $Y_{00} = 1/\sqrt{4\pi}$.

   (a) Beginning with the expression above for $\langle T\rangle$, show that we may alternatively write

$$\langle T\rangle = \frac{\hbar^2}{M}\int_0^\infty \left(\frac{du}{dr}\right)^2 \, dr \; ,$$

   where M is the nucleon mass.

   (b) Using a square-well potential, $V(r) = -V_0$ for $r < r_0$ and $V(r) = 0$ for $r > r_0$, evaluate $u(r)$ in terms of the following parameters:

$$K = \frac{\sqrt{M(V_0 - B)}}{\hbar}, \qquad R = \frac{\hbar}{\sqrt{MB}} \; ,$$

   where $B = 2.22$ MeV is the deuteron binding energy.

   (c) Show that

$$\langle T\rangle = \frac{\hbar^2 K^2}{M}\left(\frac{r_0}{r_0 + R}\right) \; .$$

   (d) Evaluate r_0 in fm and $\langle T\rangle$ in MeV assuming that $V_0 = 38.5$ MeV. Do your values justify using the nonrelativistic Schrödinger equation for the deuteron?

2. The mass of a nucleus with Z protons and A nucleons is given approximately by the semi-empirical mass formula,

$$M(Z, A) = ZM_p + NM_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(Z-N)^2}{A} + \Delta(A) \, ,$$

where $N = A - Z$. Show that for large Z and A, the energy released when the nucleus emits an α particle is given by

$$Q_\alpha = -4a_v + \frac{8}{3}a_s A^{-1/3} + 4a_c \frac{Z}{A^{1/3}}\left(1 - \frac{Z}{3A}\right) - 4a_a \frac{(N-Z)^2}{A^2} + B_\alpha \ ,$$

where $B_\alpha = 28.3$ MeV is the α-particle binding energy. (Neglect the pairing term $\Delta(A)$ in your calculation.) Show that the formula for Q_α implies that for $A > 155$ a nucleus will be unstable with respect to α decay. (Take $Z/A = 0.41$.)