Determination of the Quadratic Slope Parameter in $\eta \to 3\pi^0$ Decay

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We have determined the quadratic slope parameter α for $\eta \to 3\pi^0$ to be $\alpha = -0.031(4)$ from a 99% pure sample of $10^6\,\eta \to 3\pi^0$ decays produced in the reaction $\pi^- p \to n\eta$ close to the η threshold using the Crystal Ball detector at the AGS. The result is four times more precise than the present world data and disagrees with current chiral perturbation theory calculations by about four standard deviations.

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The decay $\eta \to 3\pi$ violates G parity and, ignoring a small electromagnetic contribution, occurs because of the u-d quark mass difference. Any low-energy theory must predict both the rate and the spectrum of the pions correctly. This decay provides an important testing ground for chiral perturbation theory (χPT), a practical, low-energy effective theory for QCD based on the chiral symmetry of the massless QCD Lagrangian that is broken when the quark masses are included. In this theory, one performs a chiral expansion in orders of "p"; a parameter that refers either to a term in the Lagrangian containing a derivative, which involves either an energy or a momentum, or one involving the quark mass, which is proportional to the Goldstone mass squared. In this picture, the leading—$\mathcal{O}(\mathsf{p}^2)$—term of the decay amplitude explicitly exhibits the dependence on $m_d - m_u$. However, this term yields a decay width $\Gamma_{\rm th}^{(0)}(\eta^0 \to \pi^+\pi^-\pi^0) = 66(8)$ eV that is a factor of 4 smaller than the measured value $\Gamma_{\rm exp}(\eta^0 \to \pi^+\pi^-\pi^0) = 281(28)$ eV as discussed by Gasser and Leutwyler [1]. They have also calculated corrections to this lowest-order result in terms of loop graphs supplemented by higher order—$\mathcal{O}(\mathsf{p}^4)$—counterterms whose values have been determined from experimental data. The predicted rate then increases to 167(50) eV. The problem was reexamined by Kambor *et al.* [2] and by Anisovich and Leutwyler [3] who used dispersive methods, which include pion rescattering to all orders. Their result raises the predicted rate to 209(56) eV, which is in better agreement but still somewhat below the experimental value.

Besides the decay rate, one must also consider the spectrum of the pions in the final state. Since the Q value is small, the decay amplitude may be expanded about the center of the Dalitz plot [1]:

$$A(\eta \to \pi^+\pi^-\pi^0) = c_0(1 + c_1 y + c_2 y^2 + c_3 x^2 + \ldots), \tag{1a}$$

$$A(\eta \to \pi^0\pi^0\pi^0) = 3c_0[1 + (c_2 + c_3)y_{\rm sym}^2 + \ldots], \tag{1b}$$

$$y = \frac{\sqrt{3}\,(s_{\pi^0} - s_0)}{2m_\eta Q_\eta}, \qquad x = \frac{\sqrt{3}\,(s_{\pi^-} - s_{\pi^+})}{2m_\eta Q_\eta}, \tag{1c}$$

$$y_{\rm sym}^2 \equiv \tfrac{1}{3}(y_a^2 + y_b^2 + y_c^2) = \tfrac{1}{3}(x_a^2 + x_b^2 + x_c^2) = x_{\rm sym}^2, \tag{1d}$$

where $s_i = (p_\eta - p_i)^2$, $s_0 = M_\pi^2 + \frac{1}{3}M_\eta^2$, $Q_\eta = m_\eta - 3m_\pi$, and the subscripts a, b, and c refer to the evaluation of x and y for each of the three neutral pions. Experimentally, the Dalitz plot for the neutral final state is usually expressed as a linear parametrization in which the variable z and coefficient α can be defined in terms of Eqs. (1) as

$$|A|^2 \sim 1 + 2\alpha z, \qquad z = (x^2 + y^2)_{\rm sym} = 2y_{\rm sym}^2,$$
$$\alpha = \frac{c_2 + c_3}{2}. \tag{2}$$

The quadratic energy dependence indicated by the α term arises at $\mathcal{O}(p^6)$ and is strictly speaking outside the one loop—$\mathcal{O}(p^4)$—calculation of Gasser and Leutwyler. However, a nonzero value is expected in a dispersive calculation [2] where rescattering effects are treated to all orders. Kambor *et al.* predict values for α in the range $-(0.014-0.007)$ depending on the value of a parameter in their calculation. The value they obtain for α is correlated to the decay rate such that the higher decay rate, which is more in agreement with experiment, requires a nearly zero value for α.

There are three previous experimental determinations of α: Baglin *et al.* [4] obtained $\alpha = -0.32(37)$ based on only 192 events; the GAMS 2000 group [5] quoted $\alpha = -0.022(23)$ based on 50k events; and the Crystal Barrel Collaboration recently presented the result $\alpha = -0.052(20)$ from a sample of $9.8 \times 10^4 p\bar{p} \to \eta\pi^0\pi^0 \to 5\pi^0$ events [6]. The resulting world average is $-0.039(15)$ [7], but the nearly 40% uncertainty does not provide a strong constraint on theoretical calculations.

We have made a new precision measurement of α based on $1.9 \times 10^7 \eta$ mesons produced in a two week run in 1998 with the Crystal Ball (CB) detector at the AGS at BNL. The CB consists of 672 NaI crystals arranged in two hemispheres which cover 93% of 4π. At its center is a 10-cm-long LH_2 target surrounded by a veto barrel made up of four scintillators [8]. A momentum analyzed, 720-MeV/c pion beam incident on the LH_2 target was used to produce the η meson near the threshold of the reaction $\pi^- p \to n\eta$. The $\eta \to 3\pi^0 \to 6\gamma$ decay is detected with less than 1% background. The direct $\pi^- p \to n3\pi^0$ reaction near the η threshold has been measured by us to have a cross section less than 2 μb [9], while the $n\eta$ cross section is large ($\approx$2 mb). The CB trigger consisted of the coincidence of a beam trigger with a CB signal requiring the total energy deposited in the CB to exceed 400 MeV. This large threshold was used to reduce triggering on the $\pi^- p \to n\pi^0$ reaction. The neutral trigger required an anticoincidence from the veto barrel surrounding the target. A beam scintillator downstream of the target vetoed CB triggers which were coincident with a noninteracting beam particle.

The process $\pi^- p \to n\eta \to n3\pi^0 \to n6\gamma$ was identified by analyzing the neutral, six-cluster events detected in the CB. The neutral clusters required a 17.5 MeV software threshold and were analyzed as photons. The analysis assumed that neutrons from η production did not produce clusters in the CB since, near threshold, about 99% of all of these neutrons passed undetected through the downstream aperture of the CB detector. Some neutrons will interact in the downstream material, and this effect was studied by Monte Carlo (MC). The kinematical hypothesis $\pi^- p \to n\eta \to n3\pi^0 \to n6\gamma$ was used to fit the events. All fifteen possible pairings of six photons to form three π^0 mesons were considered; the best pairing combination was chosen to have the lowest χ^2 value. The Z position of the π^- interaction vertex along its target trajectory was a free parameter in the fit. Events were required to satisfy the kinematic hypothesis with a probability greater than 2%, i.e., a 2% confidence level (C.L.). The value of this C.L. was adjusted as part of the systematic studies discussed below.

A sample of $1.6 \times 10^7 \pi^- p \to n\eta \to n3\pi^0 \to n6\gamma$ events was generated for the MC determination of the CB acceptance and analysis efficiency based on a full GEANT (version 3.21) simulation of the detector. For the simulation, α was set to zero. Much attention was given to reproducing the experimental efficiencies, resolutions, and the production features of the $\pi^- p \to n\eta$ reaction. The sample of events was generated for the MC using real beam events that included information of the pion's vector momentum. The Z vertex position was simulated randomly along the pion trajectory through the LH_2 target volume (the effects of beam attenuation and energy loss with Z are expected to be small). Since the cross section rises very rapidly near threshold, the change in rate across the finite dispersion of the beam was taken into account. The angular distribution for η production was simulated according to the distribution from our data. The MC efficiency response for the veto barrel has been determined experimentally from $\pi^- p$ elastic scattering [10]. The software veto-barrel threshold for the MC data was fine tuned by matching the ratio of charged/neutral triggers from the data. The hardware threshold for the CB total energy signal was also properly simulated. The experimental CB photon energy resolution $[\Delta E/E = 0.020/E(\mathrm{GeV})^{0.36}]$ was carefully reproduced in the MC. The $3\pi^0$ invariant mass distribution (formed by omitting the η mass constraint in the hypothesis) has a centroid of 547.3 MeV with a $\sigma_m = 4.9$ MeV and the MC distribution agrees with these values within 0.2 MeV for both the centroid and width, see Fig. 1a.

A data sample of 0.95×10^6 events survived analysis. The combined effect of the finite geometric acceptance, photon conversions in the veto barrel, and photon-cluster reconstruction efficiency yields an overall detection efficiency of 28%. The hardware CB energy threshold reduces it to 22.5% and smaller effects yield a final acceptance of 17.5% [11]. Figure 1b demonstrates the agreement

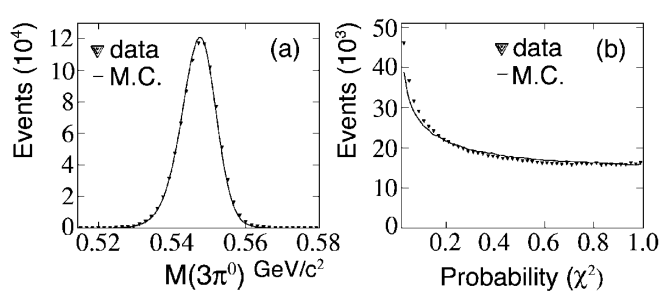

FIG. 1. (a) Comparison of the reconstructed $3\pi^0$ invariant mass distributions for data and MC. Mean and sigma values agree to within 0.2 MeV, mean is 547.3 MeV and $\sigma = 4.9$ MeV. (b) Comparison of the χ^2 probability distributions for data and MC from the standard analysis with no tunnel cuts.

between the data and MC for the probability distributions of the kinematic fit for our hypothesis. Figure 2a compares the z distributions [Eq. (2)] obtained from our data and MC ($\alpha = 0$). The bin width (0.05) was chosen to be $\approx 1.4 \times \sigma_z$. The MC z distribution is normalized to obtain a ratio of unity at $z = 0$. The shape of the MC distribution shows that our acceptance for the $\eta \to 3\pi^0$ Dalitz plot is nearly flat. Fits to the data to determine α were done for the full z distribution and also for $z \leq 0.9$. The overall variation in α for different analysis configurations is reduced significantly if we restrict our results to $z \leq 0.9$. The data for $z > 0.9$ have poor statistics due to reduced phase space (see Fig. 2a) and are more sensitive to finite resolution effects since they correspond to the edges of the Dalitz distribution. A straight-line fit to the ratio of the data and normalized MC z distributions for $z < 0.9$ is shown in Fig. 2b. The value $\chi^2/\text{ndf} = 5.2/16$ indicates good agreement with the linear-fit hypothesis. The goodness of fit precludes fitting to higher order terms.

In order to estimate the systematic error for α, we have studied in detail nine different parameters of the analysis [11]. In Table I, we compare representative results for both $0 < z < 1$ and $0 < z < 0.9$ for four of those parameters. Quoted errors for α are the fit uncertainties. Test 1 shows the effect on α of varying the minimum C.L. used for event selection. This test examines our sensitivity to unknown backgrounds. We select our final result to minimize our sensitivity to this parameter. Test 2 shows the sensitivity of α to finite bin width. We are more sensitive to this parameter when we fit only $z < 0.9$. The effect of combinatoric background can be studied by requiring only the "unique" $3\pi^0$ events. By "unique" is meant those events for which all three π^0's each reconstruct with only one valid pairing combination when requiring a χ^2 value better than the 1% C.L. (test 3). We have studied the effect of the beam entrance and exit "tunnel" crystals in the CB (test 4) by excluding events in which a cluster deposited its single-crystal maximum energy in either the first layer of crystals surrounding these "tunnels" (tunnel cut 1) or in the first or second layer (cut 2). This test is also sensitive to the effects of the recoil neutrons interacting in the tunnel regions. Although excluding these events improves the experimental resolutions, it distorts the acceptance. This distortion can amplify any differences between the MC and data and affect the value of α. This test is the only significant contribution to the systematic error when we restrict our fits to $z < 0.9$. Other parameters studied, but not shown, include the dependence of α on the $3\pi^0$ invariant mass, beam momentum, the software veto barrel threshold in the MC, the software cluster reconstruction parameters, and the beam intensity. These additional studies indicated a minimal α dependence of the order of or smaller than the first three tests in the table.

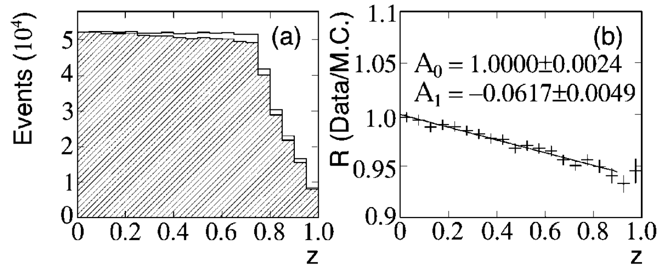

FIG. 2. Results of the test 1b analysis from Table I. (a) Comparison of the $\eta \to 3\pi^0$ density distributions obtained from data (shaded) and MC (—). A density variation with z is evident. (b) Fit to the ratio calculated from test 1b for $z < 0.9$; $\alpha = A_1/2$.

For our experimental result, we have averaged tests 1b–1d for the $z < 0.9$ fits [$\alpha = -0.0312(24)$]. Their fluctuations are consistent with the allowed statistical variation of $\alpha(\approx 0.001)$ due to changing the sample size and therefore the average can be interpreted as an essentially constant value for α within their range. Since the variations in α shown in the table represent the combined effect of both the statistical and systematic uncertainties, we take as our combined systematic and statistical error the largest variation in α due to applying the tunnel cut to

TABLE I. Different results for α as a result of varying four selection criteria (see text) used in the $\eta \to 3\pi^0$ analysis. All analyses, unless otherwise stated, are the standard analysis described in the text without tunnel cuts.

Test	Selection cuts	Event sample (%)	$\alpha(0 < z < 1)$	$\alpha(0 < z < 0.9)$
1a	χ^2 at 2% C.L.	100.0	−0.0306(22)	−0.0302(24)
1b	χ^2 at 5% C.L.	93.1	−0.0313(23)	−0.0308(24)
1c	χ^2 at 10% C.L.	84.9	−0.0321(24)	−0.0314(26)
1d	χ^2 at 20% C.L.	72.0	−0.0332(26)	−0.0315(29)
1e	χ^2 at 30% C.L.	61.3	−0.0338(29)	−0.0320(31)
2	Test 1b with 0.083 bin width	93.1	−0.0312(23)	−0.0305(24)
3a	Test 1b and unique $3\pi_0$ combination	72.4	−0.0334(26)	−0.0311(29)
3b	Test 1c and unique $3\pi_0$ combination	65.4	−0.0339(28)	−0.0319(30)
4a	Test 1b and tunnel cut 1	76.5	−0.0283(26)	−0.0275(27)
4b	Test 1b and tunnel cut 2	48.1	−0.0296(32)	−0.0283(35)
4c	Test 1c and tunnel cut 2	44.2	−0.0302(34)	−0.0284(37)

give our final result $\alpha = -0.031(4)$. No correction has been applied for the direct $\pi^- p \to n3\pi^0$ background since the value of "α" for this process would have to be ten times larger than that for $\eta \to 3\pi^0$ to alter the final result by 1σ, and such a large slope parameter is not expected theoretically. Our results determine a value for α which agrees in sign but is too large in magnitude to be accommodated by the present χPT/dispersive calculations.

Since the Kambor *et al.* result includes pion rescattering to all orders, one possible conclusion of our result is that there are dynamical effects which are contributing to the $\eta \to 3\pi^0$ decay. These terms exist in the Lagrangian, but have not been evaluated. One example of such a dynamical term is an $\mathcal{O}(\mathrm{p}^6)$ contribution to the effective chiral Lagrangian such as

$$\mathcal{L}^6 \sim \frac{F_\pi^2}{\Lambda_\chi^4} \mathrm{tr}[(\chi U^\dagger + U\chi^\dagger) D^\mu U D_\mu U^\dagger D^\nu U D_\nu U^\dagger], \tag{3}$$

where $\chi = 2B_0 m$ with B_0 a constant and m is the quark mass matrix; $\Lambda_\chi \sim 4\pi F_\pi \sim 1$ GeV is the chiral scale. The coefficients of such terms are unconstrained by the strictures of chiral invariance and are experimentally undetermined at present, since they arise at the two-loop level in the chiral expansion.

If such a dynamical contribution is present, its size should be set by chiral scale arguments via $\mathcal{A}_I \sim (m_d - m_u)/\Lambda_\chi^2 F_\pi^2$ and an isospin relation, $c_{2(\mathrm{dyn})}^{+-0} + c_{3(\mathrm{dyn})}^{+-0} = (c_2 + c_3)_{\mathrm{dyn}}^{000}$, between the $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ channels. Using the results of Gormley *et al.* [12] to fix the charged η decay parameters and subtracting them from the theoretical values obtained from the dispersive results of Ref. [2] we can determine the *dynamic* contribution to the quadratic terms: $(c_2 + c_3)_{\mathrm{dyn}}^{+-0} = -0.045(30)$, $(c_2 + c_3)_{\mathrm{dyn}}^{000} = -0.052(6)(6)$. The dynamic contribution to the linear term, c_1^{+-0}, is consistent with zero as expected since all rescattering analysis should be included in the theory already. This result is consistent with the above isospin relation, although the statistics of the charged mode prohibit a definitive test. Clearly, an improved measurement of the charged decay, together with our result, would provide a tight constraint on the χPT calculations.

In closing, we can compare our results with the related case of $K_L \to 3\pi^0$, for which a 1992 measurement [13] yielded the result: $(c_2 + c_3)_{\mathrm{exp}}^{000} = -0.020(6)(4)$. We do not have an all-orders rescattering calculation for this case as we do for the η decay, but if we assume that these effects are comparable, then there appears to be evidence for a small dynamical contribution to the kaonic decay. This result deserves further study.

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Note added in proof.—We have just become aware of a new determination of the quadratic slope parameter in $K_L \to 3\pi^0$ from [14].

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