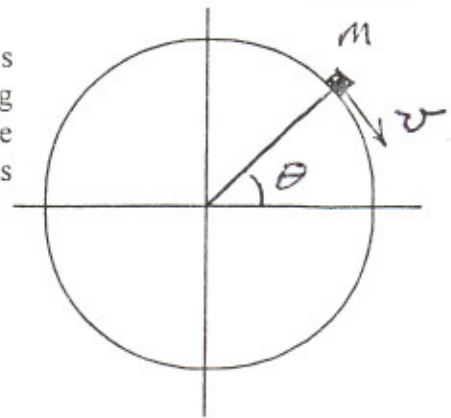


Classical Mechanics

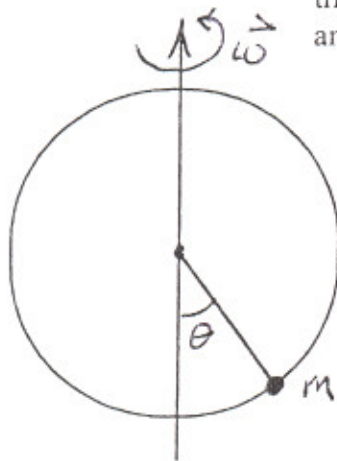
1.
35 pts.

A particle of mass m starts from rest at the top of a sphere of radius R . The particle is displaced slightly so that it slides without rolling down the sphere. Assuming no friction, at what angle θ above the horizontal will it leave the surface of the sphere and what will be its speed at that point?



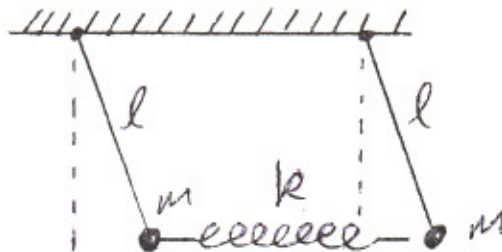
2.
30 pts.

A band of mass m slides without friction on a circular hoop of radius a . The hoop lies in a vertical plane and is constrained to rotate about a vertical diameter with constant angular velocity ω . Find the Lagrangian for this system and obtain a first integral of the motion (i.e., an expression that is a constant for this system). Is the total energy conserved? Explain.



3.
35 pts.

Consider the coupled pendulum system shown in the figure. Two simple pendulums of length l with a mass m are connected by a massless spring with spring constant k . The length of the spring is such that the spring is unstretched when the two pendula hang straight down vertically. Find the eigenfrequencies and normal modes of the system.



The system is constrained to move only in the vertical plane shown (i.e., the plane of the paper). Consider only small displacements from the equilibrium position.