Orange

Candidacy Exam   Fall 2006

Classical Mechanics

Do all problems

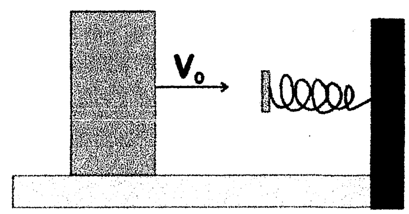
1. (35 pts.) Consider an object of mass m moving in two dimensions in a potential depending on the distance r from the origin as $U(r) = -2a_1 \dfrac{r}{r_0} + a_2\left(\dfrac{r}{r_0}\right)^2 + a_3\left(\dfrac{r}{r_0}\right)^3$ with $a_1, a_2, a_3 > 0$.

   (a) **15 pts** Find the Lagrangian, the conjugate momenta, and the Hamiltonian for this problem.

   (b) **10 pts** What quantities are conserved in this problem? Use them to show that the Hamiltonian and Hamilton's equations of motion for the non-conserved momentum can be reduced to an effective one-dimensional problem with an effective one-dimensional potential.

   (c) **10 pts** Assume that $a_2 = a_1$; $a_2 > 5a_3$ and that the initial angular momentum ℓ is such that $0 < \dfrac{\ell^2}{mr_0^2} < \dfrac{a_2}{10}$. Sketch the potential and the effective potential. Discuss the motion (bound, free, etc.) of this system for different choices of initial energies and positions. Identify on the sketch any conditions under which the motion will be circular and state (giving your reasons) whether the motion will be stable or unstable.

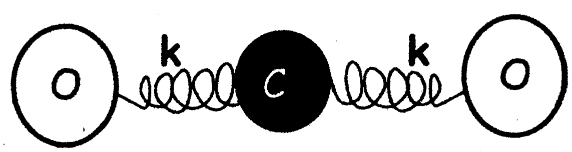
2. (30 pt.) A block of mass m slides across a frictionless table traveling at speed v_0. The block then collides with a massless spring as shown in the figure. This spring ***does not*** obey Hooke's law but produces a restoring force given by the expression $F(x) = -\alpha x - \beta x^3$. How far to the right will the mass travel after it collides with the spring and before it comes to a complete stop?



3. (35 pts.) A simple model for the carbon dioxide molecule considers it as three masses connected by springs, as shown.

   (a) (25 pts.) Assuming that all oscillations are small, find the characteristic frequencies of oscillation, considering only movement along the molecular axis. Describe the associated normal modes.

$M_O/16 = M_C/12$



   (b) (10 pts.) Carbon dioxide molecules absorb light in the infrared at ~(4.26, 7.5, and 15)$\cdot 10^{-6}$m, corresponding to a frequency ω=(4.45,2.51,1.26)$\cdot 10^{14}$s^{-1}. The first two frequencies correspond to stretching bands. Show that this is consistent with your results in (a), and calculate the force constant **k** (remember: $6\cdot 10^{23}$ oxygen atoms have a mass of $16\cdot 10^{-3}$ kg.)