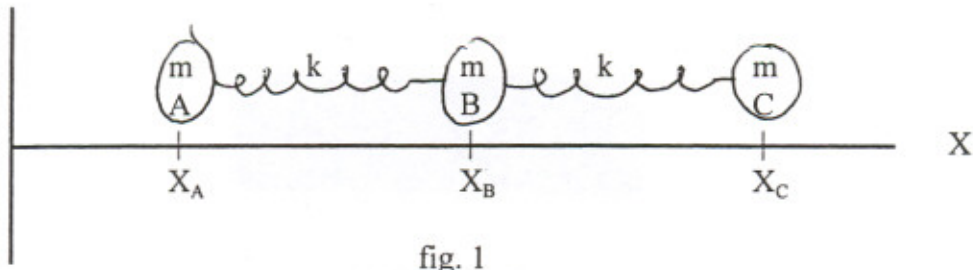


Mechanics

- (40 pts.) 1) Three identical objects of mass m are connected by springs of spring constant k ; as shown in fig. 1 below. The objects move only in one dimension.



- (10 pts.) a.) At time $t=0$, the masses are at rest in their equilibrium positions, X_A, X_B, X_C . The equilibrium length of each spring is L . Suppose the system is displaced slightly from equilibrium and then released. Write down the Lagrangian of the system, and from this deduce the equations of motion.
- (15 pts.) b.) Derive the normal modes of the system.
- (15 pts.) c.) From an initial condition where no masses are moving and the positions are such that the springs are unstretched,

$$X_A = 0, \quad X_B = L, \quad X_C = 2L$$

$$\dot{X}_A = 0, \quad \dot{X}_B = 0, \quad \dot{X}_C = 0,$$

Mass A receives an impulse

$$P = m_A \dot{X}_A \text{ at } t = 0.$$

Find the energy in each of the three normal modes as a result.

- (30 pts.) 2) Consider a rigid body of radius r and principal moments of inertia

$$I_{XX} = I_{YY} = \frac{2 m r^2}{5}$$

$$I_{ZZ} = \frac{4 m r^2}{5}$$

as determined in a body fixed frame at the center of mass. The body is simultaneously

rotating with angular velocity $\vec{\Omega}(t)$ as observed in an inertial frame. You are given $\Omega(t=0)$ and the fact that the body fixed frame coincides with the inertial frame at $t = 0$.

- (15 pts.) a.) Show that the z-component of $\vec{\Omega}(t)$ is constant.
- (15 pts.) b.) Show also that $\vec{\Omega}(t)$ precesses around the z-axis with some precession frequency ω_p . Derive an expression for ω_p in terms of $\vec{\Omega}(0)$.

- 30 pts.) 3) A uniform ladder of weight, W_1 , and length b stands on a rough horizontal plane with coefficient of friction, μ , and leans against a frictionless wall. Find the distance d that a person of weight, W_2 , can ascend the ladder without its slipping. The ladder makes an angle θ with the horizontal, and is not initially in limiting equilibrium.

