Mechanics

(40 pts.) 1) Three identical objects of mass m are connected by springs of spring constant k; as shown in fig. 1 below. The objects move only in one dimension.

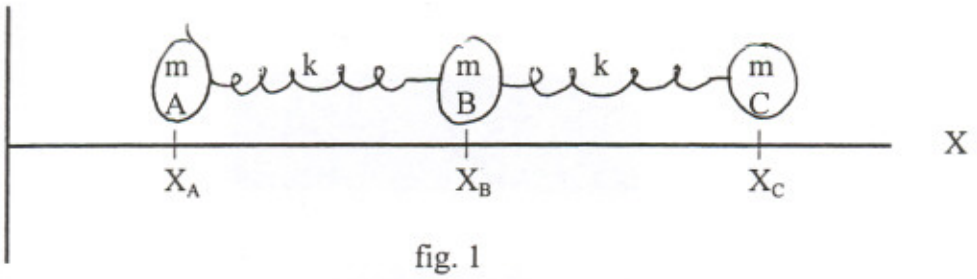

fig. 1

(10 pts.) a.) At time t=0, the masses are at rest in their equilibrium positions, X_A, X_B, X_C. The equilibrium length of each spring is L. Suppose the system is displaced slightly from equilibrium and then released. Write down the Lagrangian of the system, and from this deduce the equations of motion.

(15 pts.) b.) Derive the normal modes of the system.

(15 pts.) c.) From an initial condition where no masses are moving and the positions are such that the springs are unstretched,

$$X_A = 0,\ X_B = L,\ X_C = 2L$$

$$\dot{X}_A = 0,\ \dot{X}_B = 0,\ \dot{X}_C = 0,$$

Mass A receives an impulse

$$P = m_A \dot{X}_A \text{ at } t = 0.$$

Find the energy in each of the three normal modes as a result.

(30 pts.) 2) Consider a rigid body of radius r and principal moments of inertia

$$I_{XX} = I_{YY} = \frac{2 m r^2}{5}$$

$$I_{ZZ} = \frac{4 m r^2}{5}$$

as determined in a body fixed frame at the center of mass. The body is simultaneously rotating with angular velocity $\vec{\Omega}(t)$ as observed in an inertial frame. You are given $\Omega(t=0)$ and the fact that the body fixed frame coincides with the inertial frame at $t = 0$.

(15 pts.) a.) Show that the z-component of $\vec{\Omega}(t)$ is constant.

(15 pts.) b.) Show also that $\vec{\Omega}(t)$ precesses around the z-axis with some precession frequency ω_p. Derive an expression for ω_p in terms of $\vec{\Omega}(0)$.

30 pts.) 3) A uniform ladder of weight, W_1, and length b stands on a rough horizontal plane with coefficient of friction, μ, and leans against a frictionless wall. Find the distance d that a person of weight, W_2, can ascend the ladder without its slipping. The ladder makes an angle θ with the horizontal, and is not initially in limiting equilibrium.