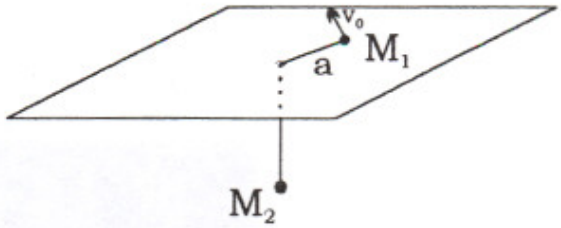
Candidacy Exam **Fall 1998**

Mechanics

Do all questions

1. (40 pts.) Mass M_1 can move freely on a horizontal table. It is connected, through a very small hole in the center of the table, with a mass M_2 that can freely move in the vertical direction only. The rope connecting M_1 and M_2 is inextensible, of length L, and can withstand a tension of T_0. Ignore friction and the mass of the rope. Take M_1 and M_2 as point masses. At time t=0, the mass is put in motion a distance a from the hole, with velocity $\mathbf{v_0}$ as shown.



(a) What is the Lagrangian of the system, in terms of independent coordinates? (For convenience later, set the potential energy U=0 for M_1 at the hole). What are the generalized momenta conjugate to your independent coordinates?

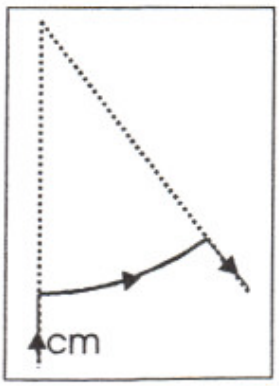
(b) Write down the Hamiltonian of the system in terms of these momenta and coordinates. Is the Hamiltonian equal to the sum of the kinetic and potential energies? Write down the canonical equations of motion. What quantities are conserved?

(c) Consider this system as a 1-dimensional problem, with an effective potential. Give the effective potential. Using this equation, make a careful, QUALITATIVE sketch, with the correct asymptotic behavior. Ignore the finite length of the string.

(d) Is a circular orbit possible (yes or no)? If so, indicate the position on the sketch. (Do NOT do any calculations!)

(e) M_1 is observed to oscillate between a maximum and minimum distance from the hole. Indicate on the sketch a possible initial TOTAL energy. For this energy, indicate on the sketch the position at which the tension T on the string would be highest (i.e. the string would be most likely to break). Explain.

2. (30pts) A child of mass m on a massless swing raises his center of mass by a small distance b every time the swing passes the vertical position, and lowers his center of mass by the same amount at each extremal position (see the sketch at right for movement of center of mass).



(a) Assuming small oscillations, calculate the work done by the child (i.e. the change in mechanical energy) per oscillation.

(b) Show that the energy of the swing grows exponentially according to $dE/dt=\alpha E$ and determine the constant α.

3. (30pts) A tall, slender, cylindrical brick chimney of height L is slightly perturbed from its vertical equilibrium position so that it topples over, rotating rigidly around its base B until it breaks at a point P (see sketch at right). Show that the most likely value for the distance d of P from B is $L/3$. Assume that the chimney breaks because the torque is too large and the chimney bends and snaps.