1) A spherical object of radius $r$ and mass $M$ rolls without slipping down a stationary ramp whose angle of inclination is $\phi$. The center of mass of the object lies at its center and the object has moment of inertia $I$ with respect to an axis through its center. Obtain the Lagrangian and solve the resulting equation of motion. Assume it starts from rest at the top of the ramp.

2) Consider the double pendulum, as shown in the figure. Find the Langrangian in terms of $\theta$, $\dot{\theta}$, $\phi$, and $\dot{\phi}$. Obtain expressions for the momenta conjugate to $\theta$ and $\phi$. Obtain the Lagrangian equations of motion and linearize for small values of $\theta$ and $\phi$.

3) An object of mass $M$ is held on a rotating turntable solely by the force of friction (coefficient of static friction $\mu$).

   a) If the turntable rotates at constant angular velocity $\omega$ and the object moves at constant speed $v$ along a radial line on the surface of the turntable, starting from the center, find the time and radial position at which friction first fails to hold the object in place.

   b) If the object is at rest a distance $r$ from the center of the turntable and the turntable has a uniform angular acceleration $\alpha$, starting from rest, find the time and angular velocity at which friction first fails to hold the object in place.