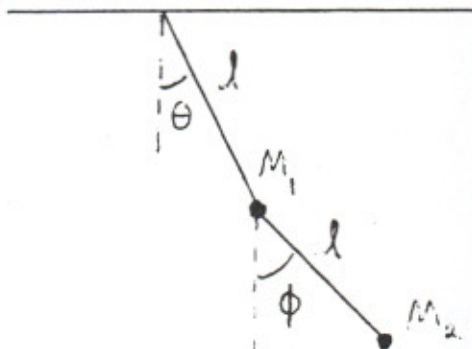


Classical Mechanics

- (30 pts.) 1) A spherical object of radius r and mass M rolls without slipping down a stationary ramp whose angle of inclination is ϕ . The center of mass of the object lies at its center and the object has moment of inertia I with respect to an axis through its center. Obtain the Lagrangian and solve the resulting equation of motion. Assume it starts from rest at the top of the ramp.

- (35 pts.) 2) Consider the double pendulum, as shown in the figure. Find the Lagrangian in terms of θ , $\dot{\theta}$, ϕ , and $\dot{\phi}$. Obtain expressions for the momenta conjugate to θ and ϕ . Obtain the Lagrangian equations of motion and linearize for small values of θ and ϕ .



- (35 pts.) 3) An object of mass M is held on a rotating turntable solely by the force of friction (coefficient of static friction μ).
- If the turntable rotates at constant angular velocity ω and the object moves at constant speed v along a radial line on the surface of the turntable, starting from the center, find the time and radial position at which friction first fails to hold the object in place.
 - If the object is at rest a distance r from the center of the turntable and the turntable has a uniform angular acceleration α , starting from rest, find the time and angular velocity at which friction first fails to hold the object in place.