Spring 2001

Candidacy Exam: Classical Mechanics

Directions: Solve all three problems.

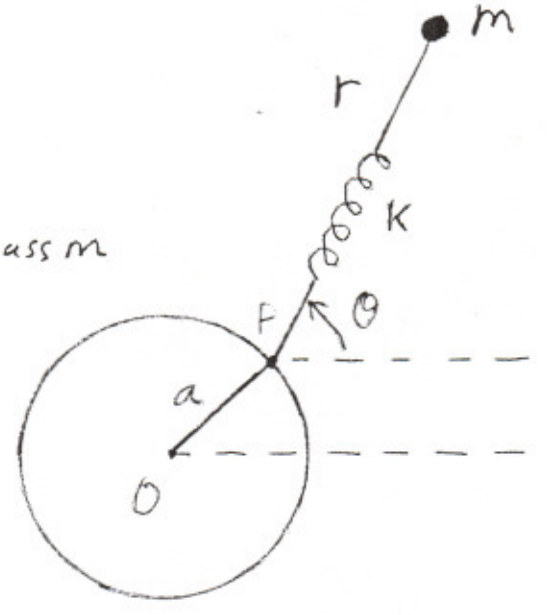
1. (25%) A bead of mass m slides without friction on a rigid circular loop of radius R. The loop lies in a vertical plane (gravity acting downward), and rotates about a vertical diameter with angular frequency ω. For what value of ω does the bead rise to a height h above the lowest point on the loop?

2. (35%) A particle of mass m is connected by a spring of stiffness constant k and unstressed length r_0 to a point P, which is moving along a circular path of radius a at a uniform angular velocity ω in a horizontal plane. (See figure below). Ignore the mass of the spring and of the apparatus connecting P to the center of rotation O.

   (a) Find the Lagrangian for this system in terms of the generalized coordinates r and θ.
   (b) Obtain Lagrange's equation for the r coordinate.

a = dist. $O \rightarrow P$

r = dist. $P \rightarrow$ mass m



3. (40%) Two identical masses M are connected by identical massless springs (stiffness k) to each other and to two rigid walls as in the figure below, which shows the situation in equilibrium. The masses rest on a horizontal frictionless surface. Calculate the displacement of each mass as a function of time t, subject to the following set of initial conditions (at $t = 0$): (1) the mass on the right is displaced by $L/2$ along the $+x$ direction from its equilibrium position, (2) the mass on the left is undisplaced, and (3) the masses have zero velocity.
Hint: Begin by calculating the normal modes of the system.