Classical Mechanics

1. (25 pts.) A pi meson ($\pi$) of mass $m_{\pi}$ at rest disintegrates into a muon ($\mu$) of mass $m_{\mu}$ and a massless neutrino ($\nu$). Show that the kinetic energy ($T_\mu$) of the muon is

$$T_\mu = \frac{(m_\pi - m_\mu)^2 c^2}{2 m_\pi}.$$

2. A particle of mass $m$ slides along a smooth, straight wire in the $x$-$y$ plane. Wrapped around the wire, and attached to the particle is a spring of force constant $k$ and unstretched length $r_0$. Both the wire and the spring are attached to the origin. The wire rotates with $\theta = \beta t$, where $\beta$ is a constant.

(\bar{v} = r \cdot \vec{e}_r + r \cdot \theta \cdot \vec{e}_\theta$ in polar coordinates.)

(10 pts.) a. Construct the Lagrangian for this system.

(10 pts.) b. Solve Lagrange's equation for this system to find the equation of motion.

(5 pts.) c. If $\beta^2 < k/m$, how would you characterize the radial motion?

3. (5 pts.) a. State Hamilton's Principle in its integral form for conservative systems.

(20 pts.) b. A body is released from a height of 64 ft and 2 sec later it strikes the ground. The equation for the distance of fall $s$ during a time $t$ could conceivably have any of the forms (where $g$ has different units in the three expressions)

$$s = gt; \quad s = (1/2) gt^2; \quad s = (1/4)gt^3$$

all of which yield $s = 64$ ft for $t = 2$ sec. Show that the integral in Hamilton's Principle is smallest for the correct one of these three forms, by calculating the integral explicitly for this time integral. You may assume $V = -mgs$.

4. (25 pts.) A particle of mass $m$, moving at relativistic speed $v$ experiences a force given by

$$\vec{F} = -\nabla V (\vec{r}).$$

The Lagrangian that gives the correct equations of motion is

$$\mathcal{L} = m c^2 \left(1 - \sqrt{1 - v^2/c^2}\right) - V (\vec{r}).$$

Use the definition of the Hamiltonian in terms of the Lagrangian to determine the relativistic kinetic energy, $T$, in terms of $m$, $v$ and $c$. 