

CLASSICAL MECHANICS

1. Pendulum (40 points)

Consider a simple pendulum of a (point) mass m suspended from a fixed point by a weightless rod of length b (moving in space subjected to a constant gravitational force).

- Find the Lagrangian as a function of generalized coordinates.
- Obtain the conjugate momenta.
- What are the constants of motion? (Explain why.)
- Use Lagrange equation to get equations of motion (leave them in differential form, do not solve them)
- Derive and sketch an effective potential V_{eff} as a function of θ (which includes the real potential U and the rotational kinetic energy of the mass moving along $\hat{\phi}$).
- Is a circular motion possible (where $\theta = \text{constant}$)? If yes, determine the frequency $d\phi/dt$.

2. Simplified Yoyo (30 points)

A string is wound around a hollow cylinder of mass m and radius R . Holding the end of the string, the body is released at height H and unwinds as he falls down.

- Derive and solve the equation of motion to get the height $h(t)$.
- What is the tension of the string during the fall?

3. Earth tunnel (30 points)

Imagine a straight channel through the center of the Earth. Consider the Earth as a homogeneous sphere with constant mass density.

- If you jump inside (lets assume with no initial speed), how long do you travel to reach the other end? (Express you answer in terms of g (gravitational acceleration constant at Earths surface) and the Earth's radius R_E)?
- What is the maximum acceleration and maximum speed?
(Do not consider any effects due to the rotation of the Earth or any resistive forces.)

