CLASSICAL MECHANICS

1. **Pendulum** (40 points)

Consider a simple pendulum of a (point) mass m suspended from a fixed point by a weightless rod of length b (moving in space subjected to a constant gravitational force).

(a) Find the Lagrangian as a function of generalized coordinates.
(b) Obtain the conjugate momenta.
(c) What are the constants of motion? (Explain why.)
(d) Use Lagrange equation to get equations of motion (leave them in differential form, do not solve them)
(e) Derive and sketch an effective potential V_{eff} as a function of θ (which includes the real potential U and the rotational kinetic energy of the mass moving along $\hat{\phi}$).
(f) Is a circular motion possible (where $\theta = constant$)? If yes, determine the frequency $d\phi/dt$.

2. **Simplified Yoyo** (30 points)

A string is wound around a hollow cylinder of mass m and radius R. Holding the end of the string, the body is released at height H and unwinds as he falls down.

(a) Derive and solve the equation of motion to get the height $h(t)$.
(b) What is the tension of the string during the fall?

3. **Earth tunnel** (30 points)

Imagine a straight channel through the center of the Earth. Consider the Earth as a homogeneous sphere with constant mass density.

(a) If you jump inside (lets assume with no initial speed), how long do you travel to reach the other end? (Express you answer in terms of g (gravitational acceleration constant at Earths surface) and the Earth's radius R_E)?
(b) What is the maximum acceleration and maximum speed? (Do not consider any effects due to the rotation of the Earth or any resistive forces.)