CANDIDACY EXAM - CLASSICAL MECHANICS - SPRING 2006

Solve all three main questions of equal weight.

1. Driven harmonic oscillator

A ball of mass m is suspended from the bottom of a massless spring (spring constant k). The opposite (upper) end of the spring undergoes forced oscillations with amplitude Y and angular frequency ω. The ball is also subject to a resistive force $F_r = -2m\beta v$ leading to an underdamped scenario with $\beta < \omega_0$, where ω_0 is the natural frequency of the undamped motion.

(a) Derive the motion of the ball (the steady state solution only). Sketch the amplitude as a function of the driving frequency ω. Determine the driving frequency ω_R where the amplitude is a maximum.
(b) For $\omega > \omega_0$ sketch the motion of the top of the spring $y(t)$, the displacement $x(t)$ and the velocity $v(T)$ of the ball illustrating the relative phase on the same time scale.

2. Rotating cube

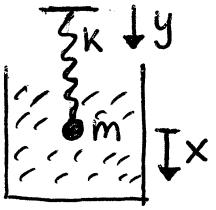
Consider a homogeneous cube (total mass M, length a) which rotates around an axis through the center of mass.

(a) What is the energy of the cube when the axis of rotation is parallel to the edge?
(b) How does the inertia moment change when the axis of rotation points along the space diagonal?
(c) Describe an experiment to measure or compare the moments of inertia.

3. Particle on cylinder

A particle of mass m is restricted to move on a cylinder surface with radius R. It is attracted by a force $\vec{F} = -k\vec{r}$ with the origin in the center of the cylinder.

(a) Find suitable generalized coordinates. Determine the Hamiltonian and Hamilton's equations in terms of suitable generalized coordinates
(b) What are constants of the motion? Explain your reasoning.
(c) Derive the motion of the particle: Find the time dependence of the location $\vec{r}(t)$ and the trajectory (relationship between the coordinates like: $x_1(x_2)$). Choose convenient initial conditions.



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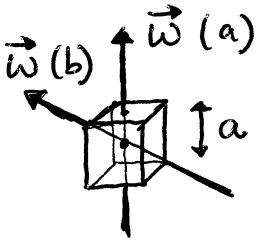

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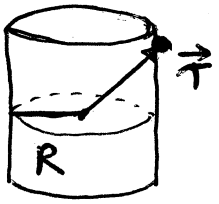

3.