Spring 2010 Candidacy Exam - Classical Mechanics

1) (20 points) In an ultra-relativistic collision, a light particle of mass m_1 and energy E_1 scatters off a stationary particle of also light mass m_2. After the collision, the stationary particle is observed to recoil at an angle θ with respect to the incident particle direction. Assuming that the collision is elastic, calculate the momentum of the recoil particle.

2) A particle of mass m is constrained to move on a cylindrical surface under the influence of a force $\vec{F} = -k\vec{r}$ where $\vec{r}$ is measured from the center of the cylinder and k is a positive constant. The base radius of the cylinder is R.

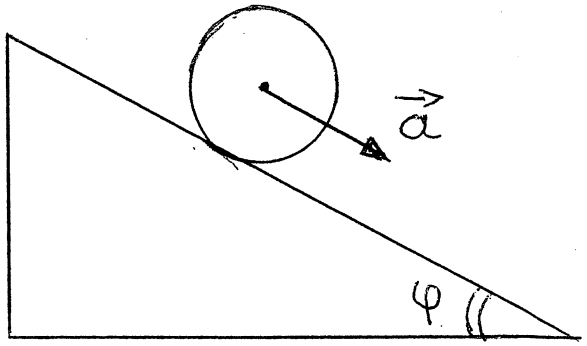
   a) (5 points) Calculate the Lagrangian and Hamiltonian of the particle.

   b) (25 points) Calculate, using Hamiltonian dynamics, the equations of motion of the particle and give an interpretation for them.

3) a) (5 points) Calculate the moment of inertia, I, of a solid cylinder with mass M and base radius R with respect to its axis.

   b) (20 points) Consider the cylinder rolling down (without slipping) an inclined plane forming an angle ϕ with respect to the horizontal direction. Calculate, using Lagrangian dynamics, the acceleration of the center of mass of the cylinder.

   c) (5 points) Use Newtonian dynamics to calculate all forces exerted on the rolling cylinder.



4) A very thin rod of mass m and length l is rotating with fixed angular velocity $\vec{\omega}$, as shown in the Figure. The angular velocity vector lies on the y-z plane of a rectangular coordinate system attached to center of the rod and makes an angle θ with respect to the rod.

a) (7 points) Calculate the inertia tensor of the rod.

b) (7 points) Calculate the angular momentum of the rod.

c) (6 points) Calculate the kinetic energy of the rod.