

## E &amp; M

Answer any 4 questions. All questions are worth 25 points.

1. A particle of charge  $Q$  is located at the origin at all times. If a spherical conducting shell of inner radius  $R$  and thickness  $\Delta R$  is assembled with the charge at its center, does the total energy of the system increase or decrease after the shell is put in place? How much work is done? [Hint: consider the energy in the electric field. Note that an alternative formulation of the same problem would ask you to find the work done in moving a charged particle from infinity to the center of the spherical conducting shell via a small hole in the shell.]
2. A rectangular loop moves away at speed  $v$  from a long straight wire carrying a steady current  $I_0$ . Two sides of the loop, of length  $w$ , are parallel to the long wire, and the other two sides have length  $h$ . The loop and wire lie in the same plane. At time  $t$ , the near side of the loop is positioned a distance  $s(t)$  from the wire. Draw a diagram which includes labeled directions for the current  $I_0$ , the direction of motion of the loop, and the current in the loop. Calculate the induced emf in the loop.
3. An ideal parallel plate capacitor has circular plates of radius  $R$  and separation  $d$ . The capacitor is located in a vacuum, and is being charged at constant current,  $I_0$ . Use the Poynting vector  $\mathbf{E} \times \mathbf{B}/\mu_0$  to show that the electromagnetic energy flux is consistent with the rate of increase of the electrostatic energy in the region between the plates, and is also consistent with  $I_0 V$ , where  $V$  is the voltage between the plates.
4. (i) [15 points] Explain the idea of a gauge transformation in the context of the electromagnetic vector potential  $\mathbf{A}$  and scalar potential  $V$ . What are the Coulomb and Lorentz gauges?  
(ii) [10 points] Determine the configuration of charges and currents that results in a vector potential

$$\mathbf{A}(\mathbf{r}, t) = -\frac{Qt}{4\pi\epsilon_0 r^3} \mathbf{r}$$

and a scalar potential  $V(\mathbf{r}, t) = 0$ .

5. Starting from the electromagnetic field tensor, find the expressions connecting the three components of an electric and a magnetic field in different inertial frames. If a positive point charge moves at speed  $\beta \sim 0.9$  along the first spatial axis ( $x_1$ ), roughly sketch the electric field as observed in the  $x_1 - x_2$  plane and in the  $x_2 - x_3$  plane. (For reference, show the electric field for the case of an observer at rest relative to the same charge.)

Possibly useful information:

$$F^{0i} = E_i/c, \quad i = 1, 2, 3; \quad F^{12} = B_3, \quad F^{13} = -B_2, \quad \text{and} \quad F^{23} = B_1$$

$$\Lambda_0^0 = \Lambda_1^1 = \gamma \quad \text{and} \quad \Lambda_2^2 = \Lambda_3^3 = -\gamma\beta$$