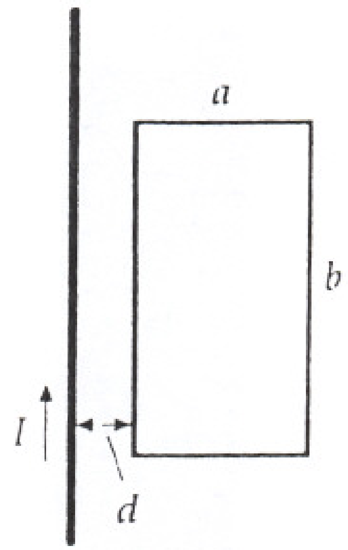
Candidacy Exam Fall 2004

Electricity and Magnetism

Instructions: Work each of the following three problems. Each problem will be graded for equal credit.

1. A long straight wire carries a current I. A rectangular loop with two sides parallel to the straight wire has sides a and b, with its near side a distance d from the straight wire, as shown in the diagram.



   (a) (20 pts) Compute the magnetic flux through the rectangular loop. (*Hint:* Calculate the flux through a strip of area $dA = b\ dx$ and integrate from $x = d$ to $x = d + a$.)

   (b) (5 pts) If the current through the long straight wire is changing at a rate $\dot{I} = dI/dt$, and the resistance of the wire in the rectangular loop is R, determine the induced current $i(t)$ in the loop.

2. (a) (8 pts) State Maxwell's equations (in vacuum) in a consistent set of units.

   (b) (8 pts) Derive from these equations the equation of continuity for the charge and current densities.

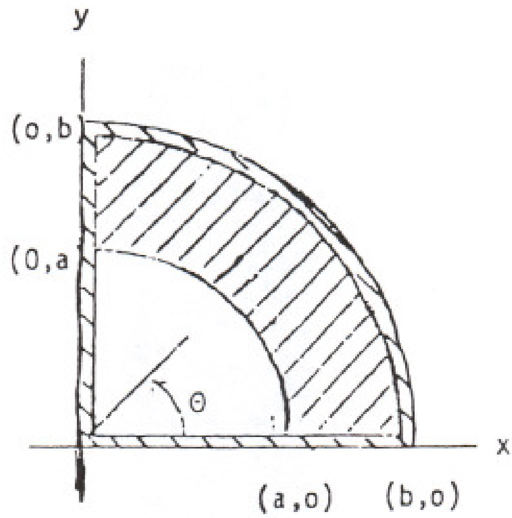
   (c) (9 pts) Derive the wave equation for the vector potential in the Coulomb gauge. Also show the scalar potential is a solution of the Poisson equation in the Coulomb gauge, $\nabla \cdot \mathbf{A} = 0$.

3. (a) (13 pts) In terms of the usual polar coordinates, (r, θ), the two-dimensional Laplace equation for the electrostatic potential Φ is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0 .$$

Use the method of separation of variables to obtain the general solution to this equation.

(b) (12 pts) Consider a very long cylindrical pipe whose cross section is a quarter-circle of radius b as shown below. For $0 \leq r \leq a$, the pipe is



evacuated while for $a \leq r \leq b$ the pipe is filled with a linear, uniform, homogeneous dielectric of permittivity ϵ. On both flat surfaces of the pipe $\Phi = 0$, while on the curved surface, $\Phi(b, \theta) = \Phi_0 \sin 4\theta$. Use boundary conditions to find $\Phi(r, \theta)$ everywhere inside the pipe. That is, find a complete set of linear equations from which one may solve for the unknown nonzero coefficients of the basis functions. *Do not solve these equations to determine the coefficients explicitly.*