E & M

For full credit, you must answer all questions.

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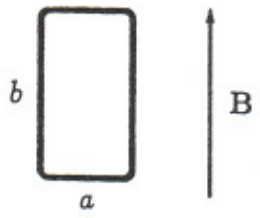
1. (*a*) (9 points) A changing magnetic field $\mathbf{B}(\mathbf{r},t)$ points in the z direction throughout the $z=0$ plane for all t. Furthermore, $\mathbf{B}$ is zero for $x^2+y^2 \geq R^2$. If $\oint_L \mathbf{E}\cdot d\mathbf{l} = \varepsilon(t)$ for a path L specified by $x^2+y^2=R^2$, write down $\oint_{L'} \mathbf{E}\cdot d\mathbf{l}$ in terms of $\varepsilon(t)$, where the path L' is specified by $x^2+y^2=4R^2$.

   (*b*) (8 points) Repeat (*a*) above for the case where the path L' has been rotated through a small angle $\delta\theta$ about *any* line in the $z=0$ plane passing through the origin.

   (*c*) (8 points) Repeat (*b*) above for the case where $\mathbf{B}(t)$ still points in the z direction but now is uniform throughout the $z=0$ plane.

2. (25 points) Two unknown but identical charges, separated by a distance d, have a grounded spherical conductor placed midway between them. What should be the approximate radius of the sphere in order for the two unknown charges to experience no net force?
   Hint: for a charge q at a distance $d/2$ from the center of a grounded conducting sphere of radius R, the image charge has magnitude $-2Rq/d$ and is located a distance $2R^2/d$ from the center of the sphere.

3. (25 points) A piece of material with conductivity σ and charge density ρ has the shape of a rectangular box, with sides a, b and c. An external voltage V is applied across the two faces which are a distance a apart. A magnetic field B is normal to the two faces which are a distance b apart. Find an expression for the Hall voltage V_H in terms of the quantities defined above.



4. (*a*) (5 points) Define phase velocity and group velocity in the context of waves.

   (*b*) (10 points) Calculate the phase velocity and the group velocity of an electromagnetic wave propagating through a nonconducting medium, assuming a simple classical model: if there are N molecules per unit volume, each having M resonant frequencies ω_j, then the permittivity and permeability of the medium can be written

$$\epsilon = \epsilon_0 + \frac{Ne^2}{m}\sum_{j=1}^{M}\frac{1}{\omega_j^2-\omega^2} \quad \text{and} \quad \mu = \mu_0\,,$$

   where e is the charge and m is the mass of the electron. This expression is a reasonable approximation if the driving frequency ω is not close to ω_j, allowing damping to be neglected. You may assume that the magnitude of the second term for ϵ is small compared with ϵ_0.

   (*c*) (10 points) Show that the phase velocity can exceed c, but the group velocity cannot. Comment on the significance of a phase velocity above c.