E & M

For full credit, you must answer all questions.

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1. A long cylindrical conducting shell with inner radius r_1 has a concentric conducting wire of radius r_2 inside. A voltage V_0 is applied between the conductors, which are separated by a dielectric gas.

   (*a*) (10 points) Derive an expression for the electric field between the conductors in terms of the above quantities.

   (*b*) (10 points) What is the time taken for ionization products in the gas to drift from radius r_a to radius r_b in terms of charge mobility μ, defined as $\mu = (1/E)\, dr/dt$?

   (*c*) (20 points) Consider the example of a chamber of the type above, 1 m long, with $r_1 = 10^{-2}$ m, $r_2 = 10^{-4}$ m, $V_0 = 2$ kV, and filled with argon gas (electron mobility $\mu_- = 0.6$ m^2/V/s and positive ion mobility $\mu_+ = 1.3 \times 10^{-4}$ m^2/V/s). The standard approach for sensing the passage of an ionizing particle through the gas is to record any voltage pulse across a resistor in series with the voltage source. (i) Draw a rough sketch of the apparatus, and estimate what condition the resistor R should satisfy in order for the chamber to generate a relatively large signal based on (ii) the electrons from ionization, and (iii) the Ar^+ ions from ionization? **Hint:** The ionization results in a current flow for a short time, and depending on the characteristic time constant of the RC circuit formed by the chamber capacitance and the resistor, this current may or may not result in a detectable voltage pulse across the resistor. The permittivity of argon gas is essentially the vacuum value $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

2. (*a*) (10 points) What is the expression for the power delivered by an emf device (e.g., a battery) with emf ε to an external load with resistance R, if the internal resistance of the emf device is r? (Be sure to express your answer in terms of ε, R and r).

   (*b*) (10 points) Beginning from this expression, find the condition that R should satisfy for the maximum possible power to be delivered to it from a battery with fixed ε and r.

3. (20 points) A static magnetic field is given, in Cartesian coordinates, by

$$\mathbf{B} = \frac{B_0}{r_0}(x\mathbf{i} - y\mathbf{j})$$

   where B_0/r_0 is a constant. Consider a region of free space where only this field need be considered; show that Maxwell's equations are satisfied.

4. (20 points) A thin conducting wire of radius r_0 is bent into the form of a rectangular loop of dimensions X and Y. If $r_0 \ll Y \ll X$, calculate the self inductance of this loop.