Candidacy Exam, Spring 2001
Electricity and Magnetism

1. (20 points) Magnetic Disk

   An engineer walks into your office and announces that she has created a high density magnetic disk under her NASA contract. It is about an inch (2.54 cm) in diameter and only 1 micron ($10^{-6}m$) thick. What makes this disk special is it has a total (integrated) magnetic dipole m that is comparable to an iron magnet of several cubic centimeters in volume.

   The engineer wants to know if this will be a good magnet for her regrigerator, so she wants to know the magnetic induction ($\vec{B}$) field near the center of the disk, just outside the disk. You naturally ask what the magnetization distribution is inside this disk and learn that it is completely uniform and parallel with the axis of the disk.

   Choose any coordinate system that you like, but calculate the magnetic field that the engineer wants just outside the center of the disk as well as at any point along the positive z-axis. Understanding that engineers only need answers that are correct to lowest order, treat the disk diameter d as much larger than the thickness h. In fact, you can treat it as though it were infinitesimally thin and was a magnetic dipole layer.

2. (30 points) Reflection

   A circularly polarized plane electromagnetic wave with intensity I_0 is normally incident on a perfectly conducting surface. Choose the z-axis as perpendicular to the surface and $z > 0$ being the exterior vacuum region. A circularly polarized wave is the sum of two planes waves 90 degrees out of phase.

   $$\vec{E}_I = E_0 \frac{\hat{x} + i\hat{y}}{\sqrt{2}} e^{-ikz-i\omega t},$$

   with $\omega = ck$.

   (a) (15 pts) Find the reflected wave, both electric and magnetic fields.

   (b) (15 pts) Find the surface current density, $\vec{K}(x, y)$, on the conductor, both magnitude and direction.

3. (30 points) Conservative Vector Fields

   And engineer comes into your office and tells you that he has measured the electric field in his newly designed heart defibrillator. He has a problem though, his measurements were along a single path in space in the shape of a parabola and he does not know if his **guess** for the electric field is correct. In his units, the parametric form of the path was,

$$x(t) = t\frac{R}{\sqrt{3}}$$
$$y(t) = t\frac{R}{\sqrt{3}}$$
$$z(t) = t^2\frac{R}{\sqrt{3}}$$

starting at $t = 0$ and ending at $t = 1$ and R is the radius of the measurement apparatus. Based on his measurements, the electric field is

$$\vec{E}(\vec{r}) = -6\frac{V_0}{R}\left(\frac{x^2 - y^2}{x^2 + y^2}(\hat{x} + \hat{y}) + \frac{z}{R}\hat{z}\right). \tag{1}$$

Your engineer friend claims this is the right field, but his integration of the E-field to get the potential difference does not give the correct potential, V_0, and he wants you to check is work.

(a) (20 pts) Does the integral

$$\int \vec{E}(\vec{r}) \cdot d\vec{l}$$

give the correct potential along the path described?
(You have to do the integral to answer this question.)

(b) Is the curl of the electric field correct?
(You have to do the curl to answer this question.)

4. Energy (20 pts)

A cylindrical resistor is constructed from a material with conductivity σ. It has a radius a and length L. The ends are capped with circular disks of low resistivity and can be treated as perfect conductors. Wires supply a constant current I to the end plates. Calculate the power generated by Joule heating

$$P = \int \vec{J}(\vec{r}) \cdot \vec{E}(\vec{r}) d^3r\,,$$

and show that his is equal to I^2R.