1. (10 pts.) a). Use Maxwell’s equations in vacuum and in the absence of charges and currents to show that the electric field $\vec{E}$ and magnetic field $\vec{B}$ satisfy individually wave equations of similar form.

(10 pts.) b) Assume that an electromagnetic wave with wave vector $\vec{k}$ is described by equations $\vec{E}(r,t) = \vec{E}_0 \sin(\omega t - \vec{k} \cdot \vec{r})$ and $\vec{B}(r,t) = \vec{B}_0 \sin(\omega t - \vec{k} \cdot \vec{r})$, where $\omega$ is the wave’s angular frequency. Show that $\vec{E}_0, \vec{B}_0$ and $\vec{k}$ form an orthogonal triad of vectors.

(10 pts.) c) Show that the magnitude of the wave are related by a proportionality constant.

NOTE: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$, for any vector $\vec{A}$.

2. Consider a long cylinder, surrounded by vacuum, of radius $R$ with volume charge density $\rho = kr$, where $r$ is the distance from its axis and $k$ is a constant.

(15 pts.) a) Find the electric field vector inside and outside the cylinder.

(15 pts.) b) Find the electric potential inside and outside the cylinder.

3. Consider a long straight wire of length $L$ that carries a steady current $I$.

(15 pts.) a) Find the magnetic vector potential at a perpendicular distance $r$ from the midpoint of the wire.

(5 pts.) b) Find the magnetic field vector at distances $r < L$. Your answer from part (a) may be useful.

NOTE: $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln(x + \sqrt{x^2 + a^2})$

4. A conducting sphere of radius $R$ and conductivity $\Sigma$ is at time $t = 0$ uniformly charged with volume charge density $\rho_v$. Find, as a function of time, the sphere’s:

(10 pts.) a) volume charge density

(10 pts.) b) surface charge density.