Candidacy Exam **Spring 2008**

Electricity and Magnetism

Instructions: Work problems 1 **and** 2, and **either** 3 or 4. The total credit is 100 points.

1. (a) (11 pts) State the differential form of Maxwell's equations in a vacuum.

   (b) (11 pts) Use the divergence theorem and Stokes' theorem to convert each of the equations in part (a) to integral form. Simplify your results as much as possible and define any new symbols or quantities that you introduce.

   (c) (11 pts) Consider a very thin infinite, straight wire that carries a uniform current I. Calculate the magnetic induction $\mathbf{B}$ outside the wire (magnitude and direction) produced by the current.

2. (33 pts) The electrostatic potential on a sphere of radius R is given by

$$\Phi(R, \theta) = k \sin^2 \frac{\theta}{2} ,$$

   where k is a constant and θ is the polar angle in spherical polar coordinates. There is no charge either inside the sphere or outside the sphere.

   Determine the potential $\Phi(r, \theta)$ everywhere inside and outside of the sphere.

3. Two point charges $+Q$ and $-Q$ are located at the ends of a line of length $2L$. The system rotates with a constant angular velocity ω about an axis perpendicular to the line and through its center

   (a) (8.5 pts) Calculate the electric dipole moment $\mathbf{p}$.

   (b) (8.5 pts) Calculate the magnetic dipole moment $\mathbf{m}$:

$$\mathbf{m} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J}(\mathbf{r})\, d^3r .$$

   (c) (8.5 pts) Calculate the components of the electric quadrupole tensor Q_{ij}:

$$Q_{ij} = \int (3x_i x_j - r^2 \delta_{ij})\, \rho(\mathbf{r})\, d^3r .$$

(d) (8.5 pts) What types of radiation are emitted and at what frequencies? Explain your answer.

4. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b, by a linear dielectric material of permittivity ϵ.

   (a) (17 pts) Determine the electric field $\mathbf{E}$ in all regions of space.

   (b) (17 pts) Determine the electrostatic potential Φ in all regions of space. (Assume that Φ vanishes at infinity.)

Possibly useful information:

The first few Legendre polynomials are:

$$
\begin{aligned}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= \frac{1}{2}(3x^2 - 1) \\
P_3(x) &= \frac{1}{2}(5x^3 - 3x) \\
P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 3)
\end{aligned}
$$