Electro-Magnetism Candidacy Exam - Jan. 2010

1) A charge with uniform linear density λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say that the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)

   a) (20 points) Find the potential in the region above the plane.

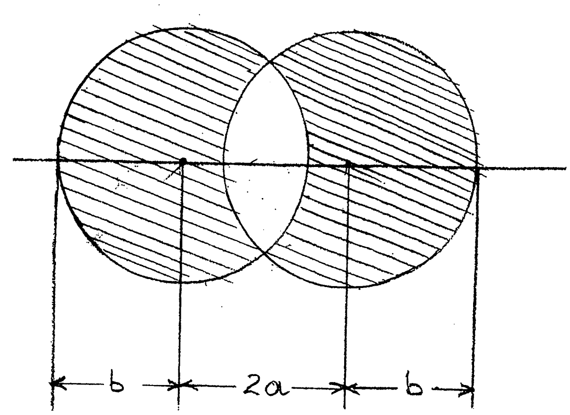
   b) (5 points) Find the charge density σ induced on the conducting plane.

2) (15 points) The electric potential of some configuration is given by the expression:

$$V(\vec{r}) = A\frac{e^{-\lambda r}}{r},$$

where A and λ are constants. Find the electric field vector, the charge density, and the total charge of the configuration.

3) (20 points) A system of cylindrical conductors has a cross section given by the intersection of two circles of radius b with centers separated by $2a$ as shown in the Figure. The conducting portion is shown shaded, the unshaded lens-shaped region being vacuum. The conductor on the left carries a uniform current density J going into the page, and the conductor on the right carries a uniform current density J coming out of the page. Assume that the magnetic permeability of the conductor is the same as that of vacuum. Find the magnetic field at all points x, y in the vacuum enclosed by the two conductors.



4) a) (10 points) Suppose that the scalar and vector potential are:

$$V(\vec{r}, t) = 0 \qquad A(\vec{r}, t) = -\frac{1}{4\pi\epsilon_\circ}\frac{qt}{r^2}\hat{r}$$

Find the fields, and the charge and current distributions.

b) (5 points) Use the gauge function

$$\lambda = -\frac{1}{4\pi\epsilon_\circ}\frac{qt}{r}$$

to transform the above potentials and comment on the result.

c) Suppose now that $V(x,t) = 0$ and $\vec{A} = A_o \sin(kx - \omega t)\hat{y}$, where $A_\circ$, k and ω are constants.

i) (5 points) Find $\vec{E}$ and $\vec{B}$.

ii) (5 points) Investigate whether $\vec{E}$ and $\vec{B}$ satisfy Maxwell's equations in vacuum.

5) a) (5 points) Show that $(\vec{E} \cdot \vec{B})$ is relativistically invariant.

b) (5 points) Show that $E^2 - c^2B^2$ is relativistically invariant.

c) (5 points) Suppose that in one inertial system $\vec{B} = 0$ but $\vec{E} \neq 0$ at some point P. Is it possible to find another system in which $\vec{E} = 0$ at the point P?