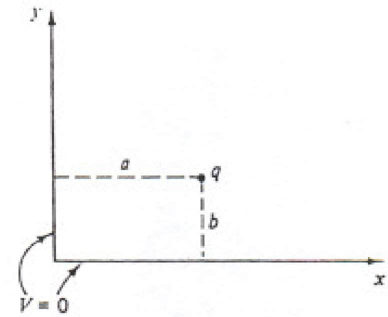
Candidacy Exam   Spring 1998

Electricity and Magnetism

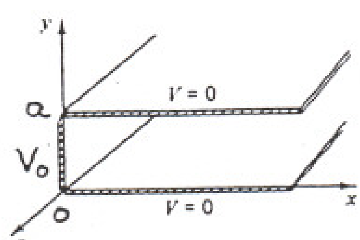
Instructions: Work each of the four problems.

1. Two semi-infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge q, situated as shown in the figure below.

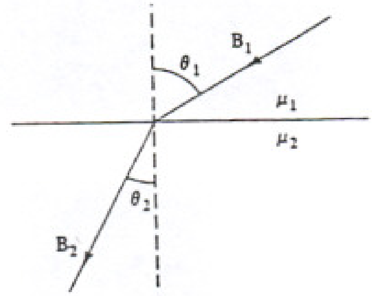


[20 pts] Use the *Method of Images* to determine the potential $V(x, y)$ explicitly in the region $(x \geq 0,\ y \geq 0)$.

2. [30 pts] Two infinite, grounded metal plates lie parallel to each other, one in the $y = 0$ plane and the other at $y = a$. The end at $x = 0$ is closed off with an infinite strip insulated from the two plates and maintained at a constant electric potential V_o. Determine the potential $V(x, y)$ inside this "slot". (*Hint*: You may express your answer as an infinite series.)



3. [25 pts] At the interface between one linear magnetic material and another, the magnetic field lines bend (see figure below). State the appropriate continuity equations, assuming that there is no free current at the boundary. Use these continuity equations to show that $\tan\theta_2/\tan\theta_1 = \mu_1/\mu_2$.



4. [25 pts] Consider a monochromatic electromagnetic plane wave propagating in free space along the direction $\mathbf{k}$. The electric field of this wave, in complex notation, is

$$\mathbf{E} = \mathbf{E}_\circ\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} ,$$

where $\mathbf{E}_\circ$ is a complex constant vector. Assume that the associated magnetic field has the same form:

$$\mathbf{B} = \mathbf{B}_\circ\, e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} .$$

Use Maxwell's equations in vacuum to determine $\mathbf{B}_\circ$ in terms of $\mathbf{k}$ and $\mathbf{E}_\circ$. Show that $\mathbf{B}$ and $\mathbf{E}$ are perpendicular to each other and to the direction of propagation. Finally, determine how the magnitude of $\mathbf{k}$ is related to ω.

Possibly useful information:

$\int_0^\pi \sin n\theta \sin m\theta \; d\theta = \frac{\pi}{2}\, \delta_{n,m}$