Candidacy Exam: Quantum Mechanics Fall 2002

(*1*) (33 pts) Answer all of the following questions:

Useful constants:

h=6.67 x 10^{-27} erg s electron mass = 9.11 x 10^{-28} g

(**a**) The smallest separation resolvable in a microscope is roughly equal to the wavelength used to view an object. What is the order-of-magnitude energy you would need to see an 0.1nm object using electrons instead of photons (i.e., in an electron microscope)? (You may give the answer in eV, J, or ergs.)

(**b**) Starting with the operator representation of x and p, prove that $[x, p] = i\hbar$ in one dimension.

(**c**) The Schrödinger equation reads

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

However, often we see

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

How are these related? How are the wavefunctions ψ in the first equation related the wavefunctions Ψ in the second version?

(**d**) Suppose the potential $V(\vec{r})$ in the Schrödinger equation has positive *parity*. In other words, $V(-\vec{r}) = V(\vec{r})$ (negative parity is when $V(-\vec{r}) = -V(\vec{r})$). What can you say about the parity of the eigenfunctions?

(**2**) (34 pts) Two-dimensional Stark effect

Suppose you have a *rotator*: two masses connected by a rigid rod. Also suppose that the rotator can only rotate in one plane (as if it is sitting on a frictionless table). Assume that its moment of inertia is ***I*** and that it has an electric *dipole moment* $\vec{p}$ directed along one direction of the rod.

(Classically, the Hamiltonian $H = \frac{L_z^2}{2I}$ where L is the classical angular momentum.)

(**a**) Write down the Schrödinger equation and solve it for the eigenfunctions and energies.

(**b**) There is a degeneracy in this problem - what is it due to physically?

(**c**) Now suppose that I turn on a constant electric field $\vec{E}$ pointing in the x-direction. Calculate the first and second order corrections to the energies found due to the perturbation caused by the electric field.

IMPORTANT NOTE: *As mentioned above, there is a two-fold degeneracy of states here. You should use* ***non-degenerate*** *perturbation theory here for the* ***second-order*** *correction, however, even though it will lead to the wrong numerical factor - the degenerate calculation is quite lengthy.*

Useful Integral: $\int_0^{\pi} e^{i(m-m')\varphi}\cos\varphi\, d\varphi = \frac{\pi}{2}[\delta_{m-m'+1} + \delta_{m-m'-1}]$

CONTINUED ON OTHER SIDE

(3) (33 pts) A spin-1/2 particle initially has $J_z = \frac{1}{2}$ (i.e., along $+\hat{z}$) at time t=0.

(a) A constant magnetic field H along $\hat{x}$ is turned on for a time t.
At the end of t, what is the probability that the spin is pointing in the $+\hat{z}$ direction?
NOTE: Don't forget to use the initial condition!

(b) Now suppose that, instead of turning on a field along the x-direction, the field $\vec{H}$ is along the $\hat{z}$-direction. What is the probability that the spin will point in the $+\hat{z}$ direction after the field has been on a time t?

(c) Finally, suppose that, at t=0, BOTH of the fields (along $\hat{x}$ and $\hat{z}$) are turned on for the time t. What is the wave function of the spin as a function of time?

Useful info: Pauli Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$