

Quantum Mechanics Candidacy Exam Fall 2007

30 pts.

1. The Hamiltonian of a system has two eigenstates Ψ_1 and Ψ_2 , with energy eigenvalues E_1 and E_2 respectively. Operator A (corresponding to an observable quantity) has two eigenstates ϕ_1 and ϕ_2 with respective eigenvalues a_1 and a_2 . Also $\Psi_1 = 3/5 \phi_1 + 4/5 \phi_2$ and $\Psi_2 = 4/5 \phi_1 - 3/5 \phi_2$. At $t = 0$, the energy is measured and the result E_1 is obtained.
- What is the state of the system?
 - At $t = t_1$, A is measured. What answers are possible, and what is the probability of each answer?
 - At $t = t_2 > t_1$, the energy is again measured. What results are possible, and what are their probabilities? Assume that the result of measuring A was not known.
 - The same as part (c), except that the result of measuring A was a_1 .

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2. Consider a one dimensional system with $V(x) = -V_0 a [\delta(x+a) + \delta(x-a)]$ where V_0 and a are positive constants. Bound states of the potential have energy less than zero. Obtain expressions for the wave functions for all bound states, using the boundary conditions to simplify as much as possible. Do not bother to find the constant necessary to normalize the wave functions. How many bound states are there?

** Comments that may be useful:

Transcendental equations may be analyzed graphically.

The function $g(z) = z[1 - \coth(z)]$, for $z \geq 0$, is monotonically increasing and $g(z=0) = 1$.

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3. The spin operator for a spin $1/2$ particle is $\vec{S} = \frac{\hbar}{2} \vec{\sigma}$ where $\vec{\sigma} = \sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z}$ and $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices:
- $$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
- using the conventional basis states of spin up and spin down along \hat{z} . The operator for spin along the \hat{n} direction, $\hat{n} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$, is $S_n = \vec{s} \cdot \hat{n}$. Find the eigenvalues and eigenvectors of S_n .