Candidacy Exam — Fall 2008

QUANTUM MECHANICS

Total: 100 Points

*** Attempt all the problems. Note that there are several parts to each problem with unequal weights, as shown in parentheses.

1. 30 points – "Quantum Land"!

In a strange "Quantum Land", the Planck's constant $\hbar = 10^4$ erg-sec, and hence has a value very different from the usual one. Assume that, otherwise, the world in the "Quantum Land" looks more or less the same as we are used to. Melons with very hard peel are grown here; they have a diameter of approximately 20 cm, and contain seeds with a mass around 0.1 gm. The average mass density of a melon is 1 gm/cm^3.

[18] (a) Assuming that one can cut open such a melon, why does one have to be careful about the seeds when cutting open melons in the "Quantum Land" ?

[12] (b) How large is the recoil velocity of a melon on reflection of "visible photons" of wavelength 628 $\times 10^{-9}$ meters from it ? Assume the collision to be elastic, and that you may use non-relativistic kinematics.

[Note: For this entire problem, order of magnitude estimates will suffice].

2. 35 points – Thinking Qualitatively !

For this problem, you do not need to explicitly solve any equation or carry out detailed calculations. You only need to provide *qualitative* answers based on physical reasoning and basic concepts relevant to quantum problems in 1-dimension.

A particle of energy E, and mass m in one dimension $(-\infty < x < +\infty)$ is subjected to a potential of the form,

$$V = \lambda \ x, \quad (\lambda > 0)$$

[5 p] (a) Plot the potential. Is the energy spectrum continuous or discrete ?

[8 p] (b) For a given energy, E, sketch the approximate forms of the wavefunctions in different regions of x-space, showing any possible qualitative differences between them.

[10 p] (c) Now consider the case when the potential is replaced by

$$V = \lambda \ |x|, \quad (\lambda > 0)$$

and $|x|$ the absolute value of x.

Plot the potential, and discuss in what ways the answers in parts (a) and (b) are changed ?

[12 p] (d) Denote the ground stationary state of the particle in the potential in (c) by the quantum number $n = 1$, and excited states by $n > 1$.

What are the number of *nodes* associated with the states $\psi_{n=1}(x)$, and $\psi_{n=10}(x)$?
What are the *parities* of these two states ?
Sketch the *wavefunctions* $\psi_{n=1}(x)$, and $\psi_{n=10}(x)$. Be careful to show the changes in wavelength and amplitude as x varies.

3. 35 points – Good old simple harmonic oscillators !

Consider a particle of mass m and charge q in a spherically symmetric potential well $V(r)$. A uniform static magnetic field $\vec{B} = B\hat{z}$ is applied along the z-axis so that the Hamiltonian is given by

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}/c\right)^2 + V(r)$$

where $\vec{p}$ is the linear momentum of the particle, and $\vec{A}$ the vector potential. The Coulomb gauge ($\vec{\nabla} \cdot \vec{A} = 0$) is chosen, and the components of $\vec{A}$ are given by:
$A_x = -By/2$, $A_y = Bx/2$, $A_z = 0$.

[15] (a) Is any component of angular momentum $L_i (i = x, y, z)$ of the particle conserved in the Hamiltonian ? Show how you reach your conclusion.

[20] (b) Assume that the magnetic field is *weak* so that terms of order B^2 in the Hamiltonian can be ignored.
Find the general solution for the expectation value $< L_x >$ as a function of time t. The expectation value is taken with respect to the quantum numbers that can be assigned to the energy eigenstates of H in the weak-field approximation.

Hint: In considering time rate of change of the $< L_i >$'s, you may want to make use of Ehrenfest's Theorem:
The time rate of change of an operator Ω is given by:

$$\frac{d}{dt} = \frac{-i}{\hbar} < [\Omega, H] >,$$

where H is the Hamiltonian of the system.