Quantum and Atomic Physics

1. Consider a Bohr orbit with a very large radius, say \( r_B = 1 \text{ mm} \), in a hydrogen-like atom with \( Z = 25 \). [Note: \( E_n = \mathcal{R} \frac{Z^2}{n^2} \), where \( \mathcal{R} = 13.6 \text{ eV} \), and the first Bohr radius of hydrogen is \( a_0 = 5.29 \times 10^{-9} \text{ cm} \).]

(10 p) (a) Estimate the corresponding quantum number, \( n \).

(10 p) (b) Assume that a transition between two neighboring quantum states occurs in this energy region (\( \Delta n = 1 \)). Calculate the energy of the emitted radiation.

(25 p) 2. Let \( |a'\rangle \) and \( |a''\rangle \) be the properly normalized eigenkets of a Hermitian operator \( \hat{A} \) with eigenvalues \( a' \) and \( a'' \), respectively (\( a' \neq a'' \)). The Hamiltonian is given by

\[
\hat{H} = |a'\rangle \delta \langle a''| + |a''\rangle \delta \langle a'|,
\]

where \( \delta \) is a real number. Determine the eigenvalues and eigenstates of the Hamiltonian.

3. Consider a beam of neutrons, each of which has momentum \( \hbar k_0 \). The beam is “chopped,” producing the following wave function for each neutron at the instant after the preparation of the chopped beam \( t_0 = 0 \):

\[
\psi(x) = \begin{cases} 
\frac{1}{\sqrt{L}} \exp(i k_0 x) & \text{if } -\frac{L}{2} \leq x \leq +\frac{L}{2} \\
0 & \text{elsewhere} 
\end{cases}
\]

The momentum of a neutron is now measured.

(15 p) (a) What values can be found and with what probability do these values occur?

(10 p) (b) Sketch the probability density as a function of the wave number. What momentum values have zero probability to be found?

(10 p) (c) What is the wave function at a later time, \( t \)? What are the probabilities of the different momentum values to be measured at time \( t \)?
(20 p) 4. At a given instant of time, the angle-dependent part of the wave function of a
particle is known to be
\[
\chi(\theta, \phi) = \left( \frac{3}{4\pi} \right)^{1/2} \sin \phi \sin \theta .
\] (3)

What possible values of \( \hat{L}_x \) and \( \hat{L}_z \) will measurement find, and with what probabilities will
these values occur? [The first few spherical harmonics are given by
\[
Y_0^0 = \left( \frac{1}{4\pi} \right)^{1/2} \\
Y_1^0 = \frac{1}{2} \left( \frac{3}{\pi} \right)^{1/2} \cos \theta \\
Y_1^1 = -\frac{1}{2} \left( \frac{3}{2\pi} \right)^{1/2} \sin \theta e^{i\phi} \\
Y_1^{-1} = \frac{1}{2} \left( \frac{3}{2\pi} \right)^{1/2} \sin \theta e^{-i\phi} .
\]