# QUANTUM MECHANICS

Attempt all the problems. There are several parts to each problem with unequal weights.

**1.** [35 points]

Suppose that a particle of mass M bounces elastically between two infinite plane walls separated by a distance D. The particle is in its lowest possible energy state.

(a) What is the wavefunction for this quantum state? Determine the energy of this state?

(b) Now assume that the separation between the walls is *adiabatically* (i.e. in an arbitrarily slow fashion) increased to 2D. Deduce the change in the expectation value of the energy of the particle.

(c) Finally assume that the separation between the walls is *suddenly* increased to 2D.

(i) Determine what happens to the energy expectation value of the particle in this case.

(ii) Compute the probability that the particle is left in the lowest possible energy state.

[*Useful Integral:*]

$$\int dx \quad \sin ax \ \sin cx = \frac{\sin(a-c)x}{2(a-c)} - \frac{\sin(a+c)x}{2(a+c)}$$

**2.** [35 points]

Consider two non-identical spin-1/2 nucleons in 3-dimensions with relative orbital angular momentum $L$. The system Hamiltonian is given by:

$$H = \frac{P^2}{M} + V$$

with

$$V = V_1(r) + V_2(r)\,\vec{\sigma_1}\cdot\vec{\sigma_2} + V_3(r)\,\vec{L}\cdot\vec{S}$$

where $\vec{\sigma_1}$, $\vec{\sigma_2}$ are the Pauli spin matrices for the two nucleons, $\vec{L}$ and $\vec{S}$ the orbital angular momentum and total spin operators, $r$ the separation between them, $P$ the relative momentum, and $M$ the mass of the nucleons.

(a) What are the "good" *quantum numbers* that can be assigned to the energy eigenstates? Give reasons. Give the *form* of these states (spatial and spin parts combined), together with the allowed *values* of the conserved angular momentum observables. No detailed derivations are required. [Recall that "good" quantum numbers relate to the complete set of commuting operators for a given Hamiltonian.]

(b) Show that the energy eigenvalue problem can be reduced to one-dimensional radial Schrodinger equation(s) for a particle in "effective" central potential(s). Present only the key steps; detailed derivation is not necessary.

(c) Obtain expression(s) for the "effective" central potentials (in part (b)) for the case(s) when the total angular momentum equals the orbital angular momentum $L$.

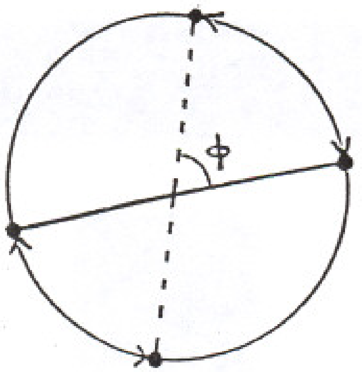
[*Useful information:*]

In spherical coordinates,

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

**3.** [30 points]

Consider two bosons of charge e, bound to the ends of a rigid rod, confined to rotate on the x-y plane. The center of mass motion is neglected. The rotation angle is $\phi$, and the moment of inertia, I.



(a) Write down (derivation not needed) the Hamiltonian, and the eigenfunctions of the bosons in terms of $\phi$. Determine the allowed energies. (Provide the reasoning needed).

(b) Now imagine that magnetic flux tubes are attached to the bosons such that they never see their own flux, but respond only to the vector potential. For the purposes of this problem, all you need to know is that the angular momentum operator of the system is redefined as follows:

$$L'_z = L_z + \frac{\theta\hbar}{\pi},$$

where $\theta = \frac{e\Phi}{\hbar c}$ determines the phase $e^{i\theta}$, $\Phi$ being the flux enclosed by a path around the circle and thus independent of the angle $\phi$.

(i) Obtain the new energy eigenvalues of the system.

(ii) Find the smallest integer choice of $\theta/\pi$ that would make the energy spectrum precisely equivalent to that of fermions. (Provide reasonings).