**Candidacy Exam** **Quantum Mechanics** **Fall 1999**

**(1)** (24 pts) These questions require either short or order-of-magnitude answers only. Do not do any exact, lengthy calculations. DO, however, show your work and the reasoning you used in obtaining your answers. You will get no credit for simply writing a number or answer down with no explanation.

Note: Planck's constant $h = 6.6 \times 10^{-27}$ erg s $= 6.6 \times 10^{-34}$ J s.

(a) Nucleons interact via the "strong force", which has a range of $\sim 10^{-13}$ cm. The mass of a nucleon is about $1.6 \times 10^{-24}$ g. Estimate, from fundamental quantum mechanical principles, the kinetic energy of a nucleon in the nucleus. Express your (very rough) answer in MeV (1 MeV $= 1.6 \times 10^{-6}$ erg $= 1.6 \times 10^{-13}$ J).

(b) Draw a picture of the wavefunction (you may choose either the real or imaginary part) of a (1-dimensional) Gaussian wavepacket for an electron with momentum $6.6 \times 10^{-27}$ gcm/s which is spatially localized in a region 4 cm in length. Label your drawing to specify any dimensions that you know.

(c) Now draw a likely qualitative picture of the same wavepacket at a later time. Assume the electron is traveling in potential free space. Explain your drawing.

(d) Why are Hamiltonian's often Hermitian?

(e) Name two physical phenomena that clearly show that classical physics is wrong. Explain why shortly.

(f) Name two aspects of the quantum solution of the simple harmonic oscillator that are distinctly non-classical.

**(2)** (25 pts) Suppose that the proton in a hydrogen atom is not a point charge but rather a homogeneous ball of charge of radius R. Then the potential is given by

$$V(r) = \begin{cases} \frac{-3e^2}{2R^3}\left(R^2 - \frac{r^2}{3}\right) & r < R \ll a_0 \\ -\frac{e^2}{r} & r > R \end{cases}$$

where $a_0$ is the Bohr radius. Calculate the perturbation in the energy of the hydrogen ground state due to this modification of the atomic potential.

*Note the condition $R \ll a_0$.* ***Just keep the absolute lowest-order approximation in quantities of order $R/a_0$ - for example, you may be able to take the exponential of such quantities to simply be equal to 1.***

Hint: You will have to think of a way to rewrite the potential so as to extract the *perturbing* potential.

**(3)** (25 pts) A Hamiltonian H obeys $H|\nu\rangle = \varepsilon_\nu |\nu\rangle$ where $\nu$ is a label for the eigenfunctions.

An *observable* A obeys $A|n\rangle = a_n |n\rangle$ for some (generally different) set of functions $\{|n\rangle\}$.

H and A are not related in any particular way (e.g., they do not necessarily commute).

(a) What is the expectation value of A for the state $\nu$: $\langle\nu|A|\nu\rangle$?

(b) What is the *probability* that, if I make a measurement of the quantity specified by A while in the state $\nu$, I get the value $a_m$?

(c) Suppose that, at time=0, I measured A to have the value $a_m$. What is the probability that I will again measure $a_m$ at a future time t?

(d) Can you think of a physical example of such a system? (Hint: read the rest of this test!)

**(4)** (26 pts) An ammonia ($NH_3$) molecule is formed of 3 hydrogen atoms which form an equilateral triangle, and a nitrogen atom that lies either above or below the middle of the triangle (see figure) – the two positions are degenerate by symmetry. Call this ground-state energy $E_0$. Assume that the N atom can only move along the z-axis, as shown in the figures. **The N atom is known to *tunnel* between the two degenerate states.**

(a) Draw a qualitative picture of the potential seen by the N atom, assuming the H triangle does not move or distort in any way. On the potential sketch a qualitative probability density for the N atom wave function.

(b) The $NH_3$ molecule has an electric dipole moment $\vec{p}$ that point in the direction of the N atom. Calculate the ground state and first excited state energies for an ammonia molecule in an electric field E along the z-axis. Express your answer in terms of the tunneling matrix element (you cannot calculate this matrix element without lots of information about $NH_3$ and a large computer!)

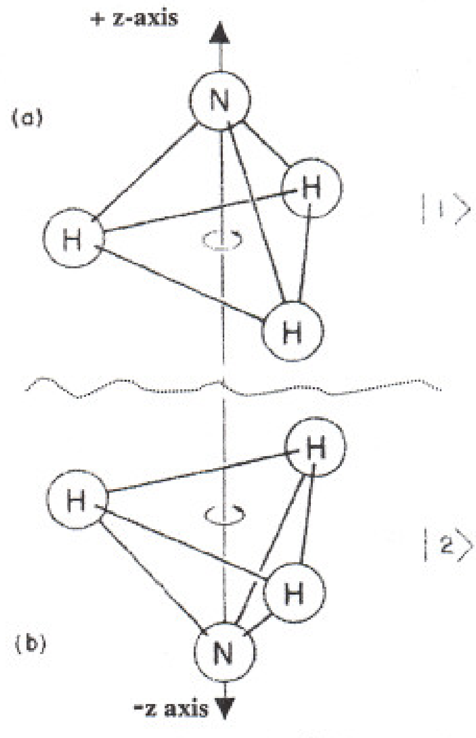


Fig. 8–1. Two equivalent geometric arrangements of the ammonia molecule.