Quantum Mechanics

1)(20pts) The following questions require only short answers, not lengthy calculations. Try, however, to be clear in your answers.

a)(5pts) Write down the expression which gives the energy levels, E_n, for the one-dimensional harmonic oscillator potential.

b)(5pts) Why can the energy in a) above never be exactly zero, i.e. why can't the particle ever be resting at the bottom of the potential?

c)(5pts) Name three effects (physical phenomena), where either particles exhibit wave-like behavior or vice-versa (these effects led to the development of Quantum Mechanics).

d)(5pts) We always say that spin is a purely quantum mechanical effect, or degree of freedom. Assume that the electron is a small sphere revolving around some axis passing through its center, and that spin is nothing else but this angular momentum. What experimental evidence from modern physics tells us that this picture is wrong?

2)(30pts) For the matrix

$$M = \begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix}$$

a)(10pts) Find the eigenvalues and the normalized eigenvectors.

b)(10pts) Can M represent a physical observable? Why?

c)(10pts) Calculate the matrix S which diagonalizes M.

3)(20pts) One of the classical concepts that was challenged in Quantum Mechanics was that of 'trajectory'. Even after the introduction of wave packets in the description of particles, physicists were trying to make them move along certain trajectories.

The time evolution of the dispersion for a one-dimensional Gaussian wave packet is (for a free particle)

$$\sigma_t = \sigma_{t=0}\sqrt{1 + \frac{\hbar^2 t^2}{4(\sigma_{t=0})^4 m^2}}$$

Let us consider the following situations:

a)(10pts) An electron in a hydrogen atom. Assuming that the initial dispersion is $\sigma_{t=0} = a_0/100$, show that even after the time of a single revolution in the Bohr model ($T \approx 10^{-16}$ sec) the wave packet will have covered the whole volume of the atom. (Order of magnitudes are fine. $m_e = 10^{-27}$ gr, $a_0 = 0.5 \times 10^{-8}$ cm, $\hbar = 10^{-27} erg \cdot sec$) Estimate the spreading of such a wave packet with initial dispersion $a_0/100$ during the time of a single orbit of a hydrogen atom. Compare this to the size of the atom.

b)(10pts) For a 'macroscopic' particle the effect is negligible. Take for example a particle with mass $m = 1mg$ and $\sigma_{t=0} = 1\mu m$. Estimate the time for a 10% change in $\sigma_{t=0}$. (One billion years $\approx 3 \times 10^{16}$ sec).

4)(30pts) Consider a one-dimensional infinite square well potential of width L containing three electrons (e^-), three positrons (e^+) and one proton. Ignore all interactions between particles.

a)(10pts) Find the ground state energy of the system.

b)(10pts) Find the energy of the first excited state of the system.

c)(10pts) Suppose we have only one particle in the well and that we put a delta function bump in the center of the well, $H' = V' = c \cdot \delta(x - L/2)$ (c is a positive constant). Find the first order correction to the allowed energies. Explain why the energies are not perturbed for even n.