QUANTUM MECHANICS

1)(15pts)

a)(5pts) The energy of a particle in an one dimensional harmonic oscillator potential can never be zero, i.e. the particle can never be found resting at the bottom of the well. In a few sentences, explain why this must be true.

b)(10pts) Show that if the operators $\hat{A}$ and $\hat{B}$ are Hermitian then the operator

$$\hat{C} = \frac{[\hat{A}, \hat{B}]}{i}$$

is also Hermitian.

2)(30pts) Consider a one-dimensional infinite square well potential of width L in the interval $(0,L)$, containing 5 electrons and one proton. Ignore all interactions between these particles.

a)(10pts) Find the normalized wavefunctions and energies for a single particle of mass m in this well.

b)(10pts) Find the ground state energy of the six particle system.

c)(10pts) Find the energy of the first excited state of the six particle system.

3)(30pts) A particle is free except that it is constrained to move on the surface of a sphere of radius R.

a) (10pts) Show that the Hamiltonian of the rotator is $H = L^2/2I$, where I is the moment of inertia and L and angular momentum.

b)(20pts) Determine the energy levels and identify the degeneracy of each.

4)(25pts) For the harmonic oscillator potential, $V(x) = (1/2)kx^2$, the allowed energies are $E_n = (n+1/2)\hbar\omega$, with $n = 0, 1, 2, ...$, where $\omega = \sqrt{k/m}$ is the classical frequency. Suppose the spring constant increases slightly, e.g. by cooling the 'spring': $k \to (1+\epsilon)k$.

a)(10pts) What are the *exact* new energies. Expand your formula as a power series in ϵ, up to second order.

b)(15pts) Write down the perturbation potential (H'). Calculate the first-order perturbation in the energy. Compare it to your result in part (a).
(Hint: You do not need to calculate any integral to do this problem. Also, you may use without proof the virial theorem for stationary states, $2 < T > = < x\frac{dV}{dx} >$).