Quantum Mechanics Spring 2010

(1) (20 pts) Is the operator $\hat{x}\hat{p}$ Hermitian? **Prove your result *without* using the fact that** $[\hat{x}, \hat{p}] = -i\hbar$!

(2) (20 pts)

Two non-interacting protons, **both with fixed spin projections m_z=+1/2**, are bound by a 3-dimensional central potential with a magnetic field in the –z direction. Assume that the radial part of the system's wavefunction is constant throughout this problem.

(a) What is the lowest energy state wavefunction (don't worry about the energy)?

(b) For this state, what uncertainty relation is obeyed by the operators $\hat{L}_x$ and $\hat{L}_y$? ($\hat{L}$ is the total angular momentum operator). (Hint – the magnetic field is just there to fix one quantum number.)

(3) (30 pts) A 1-d system is composed of a harmonic oscillator which is subject to a velocity-dependent potential:

$$V = \frac{1}{2}m\omega^2\hat{x}^2 + c\hat{p}$$

where c is a real constant.

(a) Treating the last term as a perturbation, what is the first order correction to the harmonic oscillator eigenenergies? Show your work.

(b) What are the second-order energy corrections?

(4) (30 points) A hydrogen-like wavefunction problem. Our notation for a hydrogenic atomic state is $|nlm\rangle_Z$ with energy E_n^Z, where Z is the atomic number, n the principal, l the angular, and m the z-projection of the angular momentum. Ignore spin.

An atom of tritium (an isotope of hydrogen with 2 neutrons) is initially in the state

$$(|210\rangle_1 + 2|100\rangle_1)/\sqrt{5},$$

At t=0, the tritium beta-decays, forming a one electron Helium-3 ion (2 protons, 1 neutron). The beta-decay electron and neutrino escape without depositing energy, and may be ignored.

(a) Before t=0, what was the time-dependent wavefunction of the tritium atom?

(b) For t>0, write the time-dependent wavefunction of the helium-3 ion in terms of matrix elements of tritium ($|nlm\rangle_1$) and helium ($|nlm\rangle_2$) wavefunctions. **DO NOT TRY** to compute the inner products of helium and tritium states – merely state your answer in terms of bras and kets.

(c) What is the probability of finding the helium atom in the state $|211\rangle_2$? As in (b), state your answer in terms of He and tritium bras and kets.

Useful Formulas

1-d Harmonic Oscillator

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

$$\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$$

Uncertainty Relations

$$\langle(\Delta\hat{A})^2\rangle\langle(\Delta\hat{B})^2\rangle \geq \frac{1}{4}|\langle[A,B]\rangle|^2$$