Quantum Mechanics

5 pts. 1) Consider the Zeeman effect (i.e. due to a uniform, time-independent magnetic field) on the ground state of the hydrogen atom and ignore the spin and motion of the proton.

a) Explain why the field-dependent term of the Hamiltonian can be represented by a 2x2 matrix. Give the explicit expression for this term of the Hamiltonian by choosing a quantization direction (or basis) that makes it diagonal.

b) In that basis, consider the state $|\phi>|\chi>$ where

$$|\chi> = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta} \\ e^{+i\theta} \end{pmatrix}$$

is due to the magnetic field, the position space ket $|\phi>$ is the properly normalized ground state eigenvector when the magnetic field is turned off, and θ is real. Calculate the expectation value of the total Hamiltonian in this state.

c) Find a time-dependent expression for $\theta(t)$ that makes $|\phi(t)>|\chi(t)>$ a solution of the equation of motion for this system in the presence of the magnetic field. [Here $|\phi(t)>$ is the time-dependent stationary state corresponding to $|\phi>$.]

(30 pts.) 2. What are the probabilities that measurements of L_z on the state

$$\psi(\vec{r}) = N\,(x + y + 2z)e^{-\alpha r}$$

will yield the various allowed values? [Here $\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$]

[Useful information is: $Y_0^0(\theta,\phi) = \frac{1}{\sqrt{4\pi}}$, $Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$,

$$Y_1^{\pm 1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}}\,e^{\pm i\phi}\sin\theta,\quad Y_2^0(\theta,\phi) = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1).\ ]$$

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(35 pts.) 3. The one-dimensional oscillator Hamiltonian $p_x^2/2m + \frac{1}{2} m\,\omega_x^2 x^2$ can be written as $\hbar\,\omega_x\,(a_x^\dagger a_x + ½)$ in terms of an operator a_x which is a suitable linear combination of x and p_x satisfying $[a_x, a_x^\dagger] = 1$. Use your knowledge of the properties and solutions of that system (without proof) to consider the two-dimensional system described by

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2} m\,(\omega_x^2 x^2 + \omega_y^2 y^2).$$

a) When $\omega_y = \omega_x = \omega$, what are the energies of the states $a_x^\dagger (a_y^\dagger)^2 |\phi_o>$ and $(a_x^\dagger)^2 a_y^\dagger |\phi_o>$ where $|\phi_o>$ is the ground state?

b) Determine whether or not the above two states are orthogonal and normalized. Show your work.

c) When $\omega_y = 2\omega_x = 2\omega$, what is the energy and degree of degeneracy of the 3rd excited state? Show your reasoning.

d) In terms of $a_x, a_x^\dagger, a_y, a_y^\dagger$ operators, give an orthogonal basis for the subspace corresponding to the 3rd excited state in c). Show how you know the basis you give is orthogonal.