Candidacy Exam Fall 2001

Statistical Mechanics

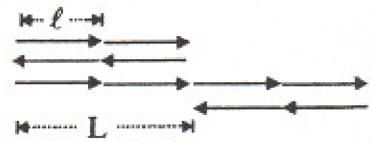
Do all problems; show all work for credit.

(1) (30 pt.) Consider a very small mirror suspended from a quartz strand with elastic constant D (i.e., the torque on the strand is given by $-D\theta$, or the energy in the strand by $\frac{1}{2}D\theta^2$), in equilibrium with a gas. The mirror reflects a laser beam in such a way that the angular fluctuations (Brownian motion) caused by the impact of surrounding molecules can be measured. The equilibrium position of the mirror is $\langle\theta\rangle = 0$. At T=287 K, the mean squared angular displacement was measured to be $\langle\theta^2\rangle = 4.20\cdot 10^{-6}$ for a strand with $D = 9.43\cdot 10^{-16}\, N\cdot m$.

(**a**; 25 pt) Calculate Avogadro's number from these experimental results. You may use the universal gas constant $R = 8.31 J\cdot K^{-1}mol^{-1}$.

(**b**; 5 pt) Can the amplitude of these fluctuations be reduced by reducing the gas density? Explain why or why not.

(2) (40 pts) Consider a very crude model of a rubber band: a chain of N links, each of length ℓ (see figure). Each link has only two possible states, pointing in either the +x or –x direction; the states have equal energy, which means equal probability. The total length L of the rubber band is the net displacement from the beginning of the first link to the end of the last link.



This chain is linked head-to-tail, as indicated by the arrow heads, and should be considered one-dimensional: the slight vertical displacement is for clarity. Note that L is the distance between the beginning of the first link and the end of t he last link.

(a; 15 pts) Find an expression for the entropy of this system in terms of N and L. It is acceptable to leave undetermined a constant that depends on N only. Hint: there are at least two acceptable approaches here:

(i) Consider the relationship between L and the total number of steps in the forward direction. If you choose this method, you may want to use the Stirling approximation, $\ln N! \approx N\ln N - N$, as well as the approximation $\ln(1+x) \approx x - \frac{1}{2}x^2 + \ldots$

(ii) Consider the probability distribution for L.

(b; 10 pts) For a one-dimensional system such as this, the length L is analogous to the volume V of a three-dimensional system, and the force f on the band is analogous to the pressure. Take f>0 for the band pulling in. From this analogy, write down an expression for the internal energy U(L,S) in terms of f, L, T, and S.

(c; 10 pts) Find the free energy F(L,T) as the appropriate Legendre transformation of U(L,S).

(d; 10 pts) Express the tension force F in terms of a partial derivative of the entropy. From this expression, compute the tension in terms of L, T, N, and ℓ.

(3) (30 pts) Consider Bose condensation for system in D dimensional space with the relation between energy and momentum given by $\varepsilon \propto |p^{\sigma}|$. Find a relation between D and σ for Bose condensation to occur.