Fall, 2002

Thermodynamics and Statistical Mechanics

Directions: Answer all three problems. Show your work; incomplete answers will receive partial credit.

1. (30%)
   (a) n moles of a diatomic ideal gas undergo an irreversible process from an initial pressure P_1 and temperature T_1 to a final pressure P_2 and temperature T_2. (Assume no molecular vibrations at these temperatures.) Find the entropy change ΔS of the gas for this process in terms of n, P_1, T_1, P_2, and T_2.
   (b) Now suppose a reversible process with $PV^{3/2}$ held constant takes the same gas between the same two states. Find ΔS of the gas in terms of n, T_1, and T_2.

2. (30%) Suppose the conduction electrons in a metal are treated as a gas of N free electrons occupying volume V at temperature T. The mean number of electrons with energy ε is given by the Fermi-Dirac distribution

$$n(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/k_B T] + 1}$$

   where μ is the chemical potential of the gas. The value of μ at $T = 0$ is called the Fermi energy, ε_F.
   (a) Sketch $n(\varepsilon)$ for $T = 0$ and $T > 0$, and label the value at ε_F.
   (b) Given a density of states $g(\varepsilon) = AV\varepsilon^{1/2}$, calculate ε_F. Express the result in terms of N, V, and the constant A.
   (c) What is the physical significance of ε_F?

3. (40%) Consider a system of N distinguishable particles with non-degenerate single particle energies given by $\varepsilon_n = n\Delta$ ($n = 0, 1, \cdots, \infty$). The kinetic energy of the particles is negligible.
   (a) Calculate the canonical partition function Z_c, assuming N is fixed.
   (b) Calculate the average (internal) energy of the system,

$$U = -\frac{\partial}{\partial \beta} \ln Z_c .$$