Thermodynamics and Statistical Mechanics

Directions: Answer all three problems. Show your work; incomplete answers will receive partial credit.

1. (30%) (a) $n$ moles of a diatomic ideal gas undergo an irreversible process from an initial pressure $P_1$ and temperature $T_1$ to a final pressure $P_2$ and temperature $T_2$. (Assume no molecular vibrations at these temperatures.) Find the entropy change $\Delta S$ of the gas for this process in terms of $n, P_1, T_1, P_2,$ and $T_2$.

(b) Now suppose a reversible process with $PV^{3/2}$ held constant takes the same gas between the same two states. Find $\Delta S$ of the gas in terms of $n, T_1,$ and $T_2$.

2. (30%) Suppose the conduction electrons in a metal are treated as a gas of $N$ free electrons occupying volume $V$ at temperature $T$. The mean number of electrons with energy $\varepsilon$ is given by the Fermi-Dirac distribution

$$n(\varepsilon) = \frac{1}{\exp[ (\varepsilon - \mu) / k_B T ] + 1}$$

where $\mu$ is the chemical potential of the gas. The value of $\mu$ at $T = 0$ is called the Fermi energy, $\varepsilon_F$.

(a) Sketch $n(\varepsilon)$ for $T = 0$ and $T > 0$, and label the value at $\varepsilon_F$.

(b) Given a density of states $g(\varepsilon) = AV \varepsilon^{1/2}$, calculate $\varepsilon_F$. Express the result in terms of $N$, $V$, and the constant $A$.

(c) What is the physical significance of $\varepsilon_F$?

3. (40%) Consider a system of $N$ distinguishable particles with non-degenerate single particle energies given by $\varepsilon_n = n\Delta$ ($n = 0, 1, \ldots, \infty$). The kinetic energy of the particles is negligible.

(a) Calculate the canonical partition function $Z_c$, assuming $N$ is fixed.

(b) Calculate the average (internal) energy of the system,

$$U = -\frac{\partial}{\partial \beta} \ln Z_c.$$