

Candidacy Exam Fall 2004
Statistical Physics and Thermodynamics

Please answer all of the questions. Show your work.

(1) A *diatomic* ideal gas has the quantum mechanical vibrational energy spectrum

$$E_n^v = \varepsilon_0 \left(n + \frac{1}{2} \right)$$

with $\varepsilon_0 / k_B = 500 \text{ K}$ ($n=0,1,2,\dots$). The scale of the rotational energy spectrum is set by the energies

$$\varepsilon_x^r / k_B = \frac{1}{2} I_{xx} \omega_x^2 / k_B = 10 \text{ K}$$

$$\varepsilon_y^r / k_B = \frac{1}{2} I_{yy} \omega_y^2 / k_B = 10 \text{ K}$$

(symmetry constrains these energies to be equal)

where the x- and y-axes are orthogonal to the line connecting the atoms (the z-axis) and I is the moment of inertia tensor.

(a) Why do we not need to specify the z-axis rotational term to compute highly accurate energies and specific heats of this gas?

(b) At 5000 K, assuming the molecules do not dissociate or become electronically excited, what is the mean thermal energy per mole of gas? What is the molar specific heat at constant volume?

(c) At 1 K, assuming the gas does not liquefy, roughly what is the molar specific heat at constant volume?

(d) Make an approximate sketch of C_p (molar specific heat at constant *pressure*) as a function of temperature from $T=0 \text{ K}$ to $T=5000 \text{ K}$. Label appropriate values of temperature and specific heat on the graph. Use a logarithmic-x axis for simplicity.

(2) Consider an ideal (that is, non-interacting) Fermi gas of N particles in a box of volume V .

(a) Find the chemical potential μ at $T=0$.

(b) Find the mean square velocity along the x-axis, $\langle v_x^2 \rangle$, at $T=0$.

(c) Calculate the mean energy \bar{E} at $T=0$. Show that, for fixed volume V , \bar{E} is NOT linearly proportional to the number of particles N . Physically, why is this?

(3) For a paramagnetic sample in a magnetic field H , the fundamental thermodynamic relation $TdS = dE + dW$ may be written as $TdS = dE - MdH$ where M is the magnetization of the sample.

(a) Suppose a sample is initially in a field H at $T=T_0$. The field is then lowered to $H=0$ *adiabatically*. Show that, during this process, we may write

$$\left(\frac{\partial T}{\partial H}\right)_S = -\frac{\left(\frac{\partial S}{\partial H}\right)_T}{\left(\frac{\partial S}{\partial T}\right)_H}$$

(b) Writing $TdS = dE - MdH$ in terms of the Helmholtz free energy $F(T,H)$, prove the Maxwell relation (do not assume an adiabatic process here)

$$\left(\frac{\partial S}{\partial H}\right)_T = \left(\frac{\partial M}{\partial T}\right)_H$$

(c) Assume the dependence of M on H ,

$$M = V\chi H$$

where χ is a field-dependent volume susceptibility. Show that

$$\left(\frac{\partial T}{\partial H}\right)_S = -\frac{VTH}{C_H} \left(\frac{\partial \chi}{\partial T}\right)_H$$

(This expression can be useful in finding the cooling that occurs as H is lowered.)

(4). Some short-answer questions:

(a) In what physical limit(s) do the Bose-Einstein and Fermi-Dirac distributions reduce to the classical Maxwell-Boltzmann distribution?

(b) Entirely in words (no math), why is it impossible to convert heat energy into mechanical energy with 100% efficiency?

(c) Part 1: Define the term "Ideal gas".

Part 2: Real gases are not ideal. Describe two ways in which real gases are not ideal.