Thermodynamics

Directions: Do problem 1 and two of the remaining three problems.

1. (40 pts) One mole of a monatomic ideal gas undergoes the following reversible cyclic process:

(a) On the adiabat $PV^\gamma$ is constant and $\gamma = 5/3$. Find the pressure $P_B$ at point $B$.

(b) Find the change in internal energy $\Delta U$ from $B$ to $C$ (i.e., on the adiabat). You may want to express your answer in units of $RT_0$, where $T_0$ is the temperature at point $A$.

(c) Find the total work $W$ done on the system, in units of $RT_0$, over one cycle.

(d) The total $\Delta S$ of the universe is zero since the cycle is reversible. However, suppose the isochor is made irreversible: the system at $A$ is suddenly placed in contact with a heat reservoir at the temperature, $T_b$, of point $B$. The other processes remain reversible. In this case, what is $\Delta S$ of the universe, in units of $R$, for one cycle?
2. (30 pts) Calculate the equation of state \( P = P(V, T) \) for a closed system whose entropy is given by \( S(U,V) = A(U/N)^{1/3} \), where \( A \) is a constant and \( N \) is the (fixed) number of particles. 
*Hint:* Use the First law of thermodynamics to obtain expressions for relevant derivatives of \( S \).

3. (30 points) Consider a typical hydrostatic system with a first order liquid – gas phase transition.

   (a) On a \( P-V \) diagram, sketch an isotherm that passes through the liquid – gas coexistence region, the critical isotherm, and a high temperature isotherm. Indicate the coexistence region with a dashed curve, and label the critical point \( C \). Put labels \( L \) (liquid), \( G \) (gas), and \( L+G \) (coexisting liquid and gas) in appropriate places.

   (b) Across the coexistence region (i.e., between points of equal pressure on either side of coexistence), which of the potentials \( H \) (enthalpy), \( A \) (Helmholtz), and \( G \) (Gibbs) of the combined \( L+G \) system change? (Recall \( H = U + PV, A = U - TS, G = U + PV - TS \).)

4. (30 points) An ideal gas contains \( N \) identical molecules, each of which is triatomic and linear. In addition to center of mass translations, each molecule has four vibrational and two rotational normal modes. Determine the heat capacity \( C_p \) of the gas, assuming all modes are activated, the atoms within the molecules interact through a harmonic potential, but there are no intermolecular interactions.