1. For a classical free particle in three dimensions obeying Boltzmann statistics, the probability that a particle is in the single particle state of energy $\epsilon_i$ is \( \exp(-\epsilon_i/k_BT) \).

(a) Write down the definition for the partition function, \( Z \), in the canonical ensemble.

(b) If the single particle energies are $\epsilon_k = \hbar^2 k^2/2m$, simplify the expression in (a) to find \( Z \). Note that $\int_{-\infty}^{\infty} dx \ e^{-x^2} = \pi^{1/2}$.

(c) Find the Helmholtz free energy \( F(T,V) \) for \( N \) particles.

(d) Find the pressure.

2. (20 pts.)

(a) Derive an exact expression for the chemical potential, $\mu$, for a two dimensional ideal Fermi gas of spin $\frac{1}{2}$ particles. Note that

$$\int dx \ (e^x + 1)^{-1} = -\ln(1 + e^{-x}).$$

(b) Evaluate the expression found in part (a) in both the high temperature and low temperature limits. Recall that $\ln(1 + x) = x$ for $|x| \ll 1$.

3. (30 pts.)

A model for a magnetic phase transition has the following Landau free energy:

$$F = \frac{t}{2}M^2 - \frac{1}{4}M^4 + \frac{1}{6}M^6,$$

where $t = (T - T_o)/T_o$. Sketch \( F(M) \) at the phase transition temperature. Obtain an expression for the equilibrium value of $M$ as a function of $t$. The phase transition occurs at what value of $t$?