Thermal/Statistical Mechanics

1. For a classical free particle in three dimensions obeying Boltzmann statistics, the probability that a particle is in the single particle state of energy ϵ_i is $\exp(-\epsilon_i/k_B T)$.

(10 pts.) a) Write down the definition for the partition function, Z, in the canonical ensemble.

(10 pts.) b) If the single particle energies are $\epsilon_k = \hbar^2 k^2/2m$, simplify the expression in (a) to find Z. Note that $\int_{-\infty}^{\infty} dx\, e^{-x^2} = \pi^{1/2}$.

(10 pts.) c) Find the Helmholtz free energy F (T,V) for N particles.

(10 pts.) d) Find the pressure.

2. (20 pts.) a) Derive an exact expression for the chemical potential, μ, for a two dimensional ideal Fermi gas of spin ½ particles. Note that

$$\int dx\, (e^x + 1)^{-1} = -\ln(1 + e^{-x}).$$

(10 pts.) b) Evaluate the expression found in part (a) in both the high temperature and low temperature limits. Recall that $\ln(1 + x) \approx x$ for $|x| \ll 1$.

3. (30 pts.) A model for a magnetic phase transition has the following Landau free energy:

$F = \frac{t}{2}M^2 - \frac{1}{4}M^4 + \frac{1}{6}M^6$, where $t = (T - T_o)/T_o$. Sketch F(M) at the phase transition temperature. Obtain an expression for the equilibrium value of M as a function of t. The phase transition occurs at what value of t?