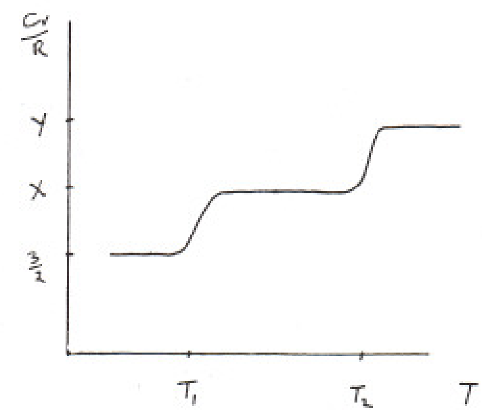
Thermal/Statistical Mechanics

(30 pts.) 1. The figure at the right shows the specific heat capacity C_v for a diatomic gas (like molecular oxygen). The axes are not drawn to scale.



(10) a) What are the values of the constants X and Y on the graph?

(10) b) What value has C_v/R for $T \to 0$?

(10) c) *Estimate* the temperature marked T_1 on the graph ($\hbar \approx 10^{-34}$ J • s, $k_B \approx 1.4 \cdot 10^{-23}$ J/K, $N_A \approx 6 \cdot 10^{23}$/mol).

(35 pts.) 2. A two dimensional, insulating solid has atoms of mass m and lattice constant a. Assume that the speed of sound c_s is the same in all directions.

(15) a) Calculate the Debye temperature T_D ($k_B T_D = \hbar \omega_D$ with maximum eigenfrequency ω_D).

(5) b) Calculate the high temperature molar heat capacity.

(15) c) Show that as the temperature approaches zero, so does the heat capacity.

(35 pts.) 3. Consider a paramagnet composed of atoms having spin 1. When these atoms are placed in a magnetic field H, the Hamiltonian can be written:

$$\mathcal{H} = -\sum_{i=1}^{N} \mu_0 H S_Z^{(i)}$$

where S_Z can take on values -1, 0, or +1.
Using any ensemble you find convenient, calculate exactly the magnetization as a function of temperature.