Thermodynamics and Statistical Mechanics

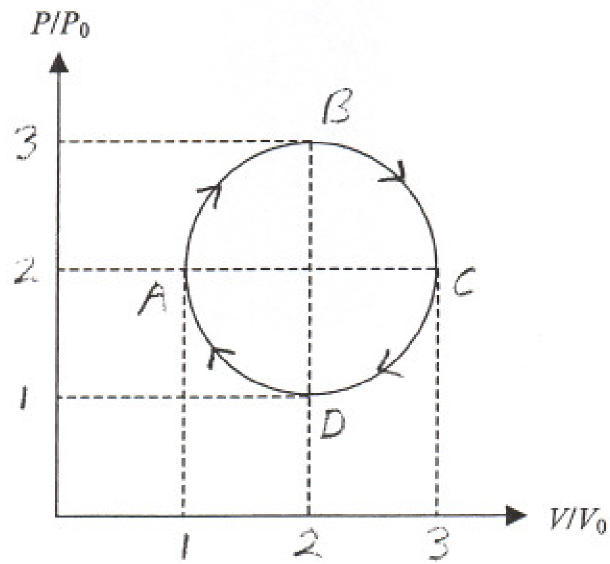
Instructions: Solve all three problems.

1. (30 points) The molar specific heat at constant volume of a monatomic ideal gas is known to be $3R/2$. Suppose that one mole of such a gas undergoes a cyclic quasistatic process which appears as a circle in the P-V plane shown below. (P_0 and V_0 are a reference pressure and volume, respectively).

   (a) Calculate the net work done by the gas in one cycle in terms of the product P_0V_0. Hint: Consider $PdV = P_0V_0 \frac{P}{P_0} d\left(\frac{V}{V_0}\right)$.

   (b) Calculate the internal energy change of the gas between state C and state A in terms of P_0V_0.

   (c) Calculate the heat absorbed by the gas in going from A to C via the path ABC of the cycle, in terms of P_0V_0.



2. (40 points) Consider a system of N localized particles with non-degenerate energy levels given by $\varepsilon_0 = 0$, $\varepsilon_1 = \Delta$.

   (a) Calculate the canonical partition function Z_C.

   (b) Calculate the average energy $\overline{E}$ of the system in contact with a heat reservoir.

(c) Given $\overline{E^2} - \bar{E}^2 = -\frac{\partial}{\partial \beta}\bar{E}$, calculate the fractional rms energy fluctuation,

$$f = \left(\frac{\overline{E^2} - \bar{E}^2}{\bar{E}^2} \right)^{1/2}$$

, of the system in terms of N, β, and Δ.

(d) Suppose $\Delta = 10^{-16}$ erg and there are 10^{23} particles. Estimate the temperature range for which $f \geq 1$. (Recall $k_B = 1.38 \times 10^{-16}$ erg/K.)

3. (30 points) Suppose the quantum state of a harmonic crystal consisting of N atoms is given by a list of $3N$ quantum numbers $\{n_1, \ldots, n_i, \ldots, n_{3N}\}$, each of which is a non-negative integer ($n_i = 0,1,2,\ldots$). The corresponding energy levels of the system are given by $\varepsilon_i = (n_i + 1/2)\hbar\omega_i$.
   (a) Calculate the mean energy of the system at temperature T.
   (b) Assume the oscillator frequencies do not depend on T. Evaluate the energy in the limit $T \to 0$ and explain why it is non-zero.