Spring 2007

Thermodynamics/Statistical Mechanics

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40 pts. 1) A certain solid can exist in two phases at low temperatures. At normal pressure (1 atm), the chemical potentials μ_i (i = 1,2) of the two phases have the form

$$\mu_i = a_i - b_i T^2 - c_i T^4, \qquad i = 1,2$$

where the coefficients $a_1 = 3.5$ J/g, $a_2 = 0.5$ J/g, $b_1 = 4$ J/gK^2, $b_2 = 2$ J/gK^2, $c_1 = 2$ J/gK^4, and $c_2 = 1$ J/gK^4.

(10 pts.) (a) Find the temperature T_c of the phase transition. Indicate which phase is stable below T_c and which phase is stable above T_c. Why?

(10 pts.) (b) Calculate the latent heat of this transition? Based on your result, is this transition first or second order? Why?

(10 pts.) (c) Calculate the specific heat jump ΔC_p at T_c.

(10 pts) (d) Assuming that the difference between the specific volumes of the two phases $V_1 - V_2 = 1$ cm^3/g is temperature independent, find at what pressure the transition temperature would be $T_c = 2$ K.

30 pts. 2) In general, the change in the internal energy of a magnetic system is given by dU = T dS + H dM, where S is the entropy, H is the magnetic field, and M is the magnetization. Consider a paramagnetic material which obeys the Curie law $\chi_T = K/T$ (K is a constant, $\chi_T = M/H$) and has a heat capacity $C_{M=0} = b/T^2$ for zero magnetization. Derive the heat capacity

(20 pts.) (a) at constant magnetization C_M and

(10 pts) (b) at constant magnetic field C_H.

30 pts. 3) A water molecule can vibrate in various ways, but the easiest type of vibration to excite is the "flexing" mode in which the hydrogen atoms move toward and away from each other but the HO bonds do not strech. The oscillations of this mode are approximately harmonic, with a frequency of 4.8×10^{13} Hz. As for any quantum harmonic oscillator, the energy levels are (1/2)hf, (3/2)hf, (5/2)hf, and so on. None of these levels are degenerate.

(15 pts.) (a) Calculate the probability of a water molecule to be in its flexing ground state and in each of the first two excited states, assuming that it is

in equilibrium with a reservoir (say the atmosphere) at 300 K. (Hint: Calculate Z by adding up the first few Boltzmann factors, until the rest are negligible.)

(15 pts.) Repeat the calculation for a water molecule in equilibrium with a reservoir at 700 K (perharps in a stem turbine).

Note: $h = 4.136 \times 10^{-15}$ eV s; $k_B = 8.617 \times 10^{-5}$ eV/K.