Physics 23102 **Fall 2001**

Gauss' Law

Coulomb's law can be derived from Gauss' law, but not vice versa, since Gauss' law is more general.

$$\epsilon_0 \Phi_E = q_{enc} \ , \qquad \Phi_E = \oint_S \mathbf{E} \cdot d\mathbf{A} \ ,$$

where the Gaussian surface S is *any* closed surface we choose to imagine, and q_{enc} is the algebraic sum of *all* electric charges enclosed by the surface S. Remember that charges *outside* S contribute to $\mathbf{E}$ but their contribution to Φ_E cancels to zero (see sample problem 24-1, page 546 in your textbook).

If our problem does not have a high degree of symmetry, Gauss' law is still true, but it doesn't help us to find $\mathbf{E}$. If symmetry *is* present, Gauss' law gives us a very easy way to calculate the electric field $\mathbf{E}$ set up by a certain distribution of charge — symmetry allows us to bring $\mathbf{E}$ outside the integral so that $\Phi_E = E \int_{S_\parallel} dA$ and then $\int_{S_\parallel} dA$ is simply the area of that part of S where the vectors $\mathbf{E}$ and $\mathbf{A}$ are parallel or antiparallel. Here are the guidelines for choosing S:

Spherical symmetry Choose S to be a concentric sphere; if we want to find $\mathbf{E}$ at a given radius r_1, then S must have that same radius r_1. We must know from symmetry that $\mathbf{E}$ has the same magnitude at all points on the spherical surface S; then $\int_{S_\parallel} dA = 4\pi r_1^2$ (the complete surface of the sphere of radius r_1).

Cylindrical symmetry Choose S to be a coaxial cylinder; if we want to find $\mathbf{E}$ at a given radius r_1, then S must have that same radius r_1. We must know from symmetry that $\mathbf{E}$ has the same magnitude at all points on the *curved* part of the cylinder S and always crosses that part at 90°. We must also know from symmetry that the end-caps of the cylinder S make no contribution to the flux Φ_E (normally because $\mathbf{E}$ lies parallel to the end-caps, and so is at 90° to the vector $\mathbf{A}$ for the end-caps). Then $\int_{S_\parallel} dA = 2\pi r_1 h$, where h is the length of the cylinder.

Planar symmetry Choose S to be a cylinder (although in this instance, various other shapes would also work). The cylinder must be oriented so that the electric field is always parallel to the curved part of the cylinder, and the field must have the necessary planar symmetry for this to be possible. Then the only contribution to the flux comes from the end-cap(s). The planar symmetry guarantees that $\mathbf{E}$ is uniform over the end-cap(s). One of the end-caps can be taken to lie in the plane in which we wish to calculate $\mathbf{E}$. The other end-cap will either contribute zero flux because it lies inside the body of a conductor (where $\mathbf{E}$ is always zero in electrostatics), or else it contributes an equal flux because of symmetry. Then $\int_{S_\parallel} dA =$area of one or both end-caps.