PHY 6/75101 October 5, 1998

Classical Mechanics — EXAM I

Each question carries the same credit.

1. A meteorite (mass m) falls vertically to the earth (mass M). Assume that the meteorite effectively started an infinite distance away with zero relative speed. Use Lagrange's equation to find the meteorite's speed as a function of distance from the center of the earth. Consider only distances beyond the earth's atmosphere.

2. Consider a *spring pendulum*: a massless spring with spring constant k, fixed at one end but free to pivot in any direction, located in a gravitational field $\mathbf{g}$ with a mass m attached to the other end. Let the equilibrium length of the spring without any mass attached be ℓ.
(a) Write down the Lagrangian for this system.
(b) Determine the conjugate momenta.
(c) Determine the equations of motion (but don't attempt to solve them).
(d) What are the constants of the motion?

3. A hoop of mass M and radius R rolls down an inclined plane without slipping. The plane makes an angle ϕ with the horizontal. There are two constraints in this problem: the constraint of the hoop being in contact with the plane, and the rolling constraint.
(a) Define a suitable set of coordinates and express both constraints mathematically. Are either or both of these constraints holonomic?
(b) Explain the physical meaning of each of the two forces of constraint.
(c) Choose a set of generalized coordinates that allows the first constraint to disappear from the problem, but not the second.
(d) Write down the Lagrangian for the hoop in terms of your chosen coordinates.
(e) Use the Lagrange Multiplier method to calculate the second force of constraint.

4. Keeping mathematical detail to a minimum, explain each of the following:
(a) Symmetry, cyclic coordinates, & conservation laws.
(b) Applications of dissipation functions.
(c) Virtual displacements & virtual work.
(Part (a) should be about half to one page; parts (b) and (c) can be much shorter.)