PHY 6/75101 November 20, 1998

Classical Mechanics — EXAM II

Each question carries the same credit.

1. The piston engine used in a light aircraft of conventional design turns the propeller clockwise, as viewed by the pilot. Consider an airplane whose propeller has four blades, each of mass m and length b, and rotating at a constant angular frequency $\dot{\psi}$. The plane executes a level left turn, and its heading changes at a uniform rate $\dot{\eta}$. What torque vector must be applied to the propeller shaft to maintain this state of motion? Describe how the direction of this torque changes with time in each coordinate system. [**Hint:** the main part of this problem resembles one you have already seen, but note that the propeller now has *four* blades, not two.]

2. Three springs are fastened end-to-end in a straight line. A mass m is attached at each end of the middle spring, and the outside end of each outer spring is fixed rigidly. The middle spring has force constant $3k$ and the other two have force constants k. All springs are at their equilibrium length when the system is at rest, and gravity need not be considered. Find the eigenfrequencies for small vibrations, and describe each normal mode.

3. The Newtonian gravitational potential for two bodies of mass M and m is often written in the form $V = -ku$, where $u = 1/r$ and $k = GMm$. For the case of negative total energy, it can be shown that the motion is an ellipse of the form

$$u = C[1 + e\cos(\theta - \theta_0)]$$

where the constants C, e and θ_0 depend on the initial conditions and are not of importance here. Beginning from the standard approach $m\ddot{r} = f'(r)$ in which the problem is effectively reduced to one dimension, consider a modified potential $V = -ku + hu^2$ where h is a constant.

(a) Show that the motion can be described by a precessing ellipse.

(b) In a homework problem assigned recently, you considered a planet orbiting a star surrounded by a low density dust cloud. There, it was found that the sign of the precessional angular frequency was opposite to that of the orbital angular frequency. What about the present case? Explain your reasoning carefully.